Examrace

CAT Model Paper 3 Questions and Answers with Explanation Part 5

Q: 26. Ravi has recently bought an android phone. It has a screen lock which operates in the following way: there are 9 dots arranged in a 3×3 square formation. Unfortunately, after setting the lock, he has forgotten it. The only thing he remembers is that to unlock, one has to tap 4 dots in the correct sequence.

What is the probability that he will open the lock in the 37^{th} trial given that he does not repeat any failed trials?

(A) $\frac{37}{3024}$

(B) $\frac{36}{3024}$

(C) $1 - \left(\frac{37}{3024}\right)$

(D) None of these

Ans: D

Solution:

The total number of sequences possible is ${}^{9}P_{4} = 3024$.

The probability of succeeding at the first trial is $\frac{1}{3024}$.

In the nth trial (assuming the first $(n-1)$ trials are unsuccessful), he will not try out the options tried out in the previous $(n-1)$ trials. Hence, the number of sequences from which he chooses one at random is 3024 – (n - 1) or $3025 -$ n. Therefore, the probability of success in the nth trial alone is $\frac{1}{3025-n}$. The probability of success in the nth trial having failed in the previous $(n-1)$ trials is $\frac{3023}{3024} \times \frac{3022}{3023} \times \frac{3021}{3022} \times \ldots\ldots \frac{(3025-n)}{(3026-n)} \times \frac{1}{3025-n}$.

Hence, the probability of success in any trial is $\frac{1}{3024}$

Option (D)

Q: 27. How many A.P.s of 3 terms can be formed from the set of the 1^{st} 40 natural numbers?

(A) 40

(B) 380

(C) 760

(D) None of these

Ans: C

Solution:

All A.P.s must have integral common difference (given that all terms are natural numbers).

Every triplet will give us 2 series, one increasing and the other decreasing e.g. $(1,\ 2,\ 3)$ & $(3,\ 2,\ 1)$.

Number of series with common difference 1, e.g., $(1,\ 2,\ 3)$, $(2,\ 3,\ 4)$, $\ldots\ldots\ldots\ldots\ldots$ $(38,\ 39,\ 40) = 38 \times 2$ (1^{st} term can be anything from 1 to 38)

Number of series with common difference $2\ =\ 36 \times 2$ (1^{st} term can be anything from 1 to 36)

Number of series with common difference $3\ =\ 34 \times\ 2$ (1^{st} term can be anything from 1 to 34)

Number of series with common difference 19 (highest common difference) $= 2 \times 2$

So, total number of series = $2 \times\ (2\ +\ 4\ +\ \ldots\ldots\ldots\ldots\ +36\ +\ 38) =\ 760$.

Q: 28. Mamata borrowed a total of Rs. $40{,}000$ from two persons to be repaid at the end of two years, with the interest being compounded annually. She repaid Rs. 31,100 more to the first person when compared to the second at the end of two years. The first person's rate of interest was 10 percentage points more than that of second. Instead, if Mamata borrowed equal amounts from them at the same rates, she would have paid Rs. $4{,}600$ more to the first. Find actually how much was borrowed from the first person.

(A) Rs. $25{,}000$

(B) Rs. $28{,}500$

(C) Rs. $30{,}000$

(D) Rs. $35{,}000$

Ans: C

Solution:

Let the interest rate charged by first and second person be $(r\ +\ 10)\%$ and $r\%$

respectively.

When Mamata borrows equal amounts from the two persons at these rates, the difference in n amount repaid is due to difference in interest calculated and equal to 4600, i.e.,

$20000\left[\left(1+\frac{r+10}{100}\right)^2-\left(1+\frac{r}{100}\right)^2\right]=4600$ Solving the equation above, we get $r=10$

Now let us consider that the loans taken from the first and second persons were x and $40{,}000-x$ respectively.

We are given that the difference between 20% on x and 10% on $40{,}000-x$ is $31{,}100$

Formulating the equation as above and solving, we get $x=30{,}000$.

Q: 29. Triangle PQR is isosceles and has sides $PQ=QR=3$ units. A line AB is so drawn such that A lies on PR and B lies on QR and is distinct from Q and it divides the triangle into two parts having equal area as well as equal perimeter. If $PR=4$ units. Find AR.

(A) 2 units

(B) 3 units

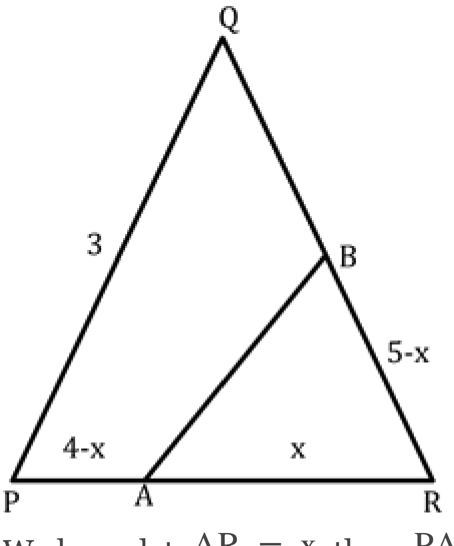
(C) 2.5 units

(D) 3.5 units

Ans: B

Solution

Consider the figure given below



We have, let $AR=x$ then $PA=4-x$. Perimeter of triangle $PQR=3+3+4=10$ units. Further, since the perimeters of ABR and APQB are the same and AB is common to both, $AR+RB=AP+PQ+QB=\frac{10}{2}\text{or }5$. Since $AR+RB=5\text{Units},\ RB=5-x$ units.

Now area of triangle $QRP=2x$ area of triangle ARB (given that the areas of ABR and $APQB$ are equal).

Thus $\frac{1}{2}AR\times RB\sin R=\frac{1}{2}\times\left(\frac{1}{2}\times QR\times PR\sin R\right)$; or,

$2(x)(5-x)=12; or,\ x^2-5x+6=0$

Solving, we get that $x = 2,3$. If $x = 2$, then B would coincide with Q which contradicts the condition given in the problem. Hence $x = 3$, i. e. $AR = 3$ as given in option (B).

Q: 30. Let us consider a set of 100 distinct positive numbers. From that set, all possible distinct combinations of 99 numbers are selected and their averages computed as $A1,\ A2,$ â€ $A100.$ Let X is the average of the complete set of averages so obtained. If Y be the average of the original set of 100 numbers, then which of the following is definitely true regarding the relationship between X and Y?

(A) $X > Y$

(B) $X < Y$

(C) $X = Y$

(D) Depends on the 100 numbers selected.

Ans: C

Solution:

Let n_i be any number present within the original set of 100 numbers. Out of the 100 possible combinations, this number definitely belongs to 99 distinct combinations. Thus if the sum of the original numbers is S, then the sum of the sums of each of the individual sets of 99 numbers would be $99S$ .Then, it is easy to see that $S_1 + S_2 + \ldots + S_{100} = 99S$, where Si is sum of the i^{th} combination of 99 numbers. Dividing both sides by 99, we obtain , $A_1 + A_2 + \ldots + A_{100} = S$; which implies that the average of the complete set of averages obtained would also be the average of the original 100 numbers. Hence $X = Y$ as given in option (3).

Alternate explanation for those who find the verbal reasoning difficult: If Ai is the average of all numbers excluding the i^{th} number (Ni) and Y is the original average of the complete set of numbers,

$100Y = 99Ai + Ni$

or $Ai = \frac{100Y - Ni}{99}$

$X = \frac{Ai}{100}$

$= \frac{(100Y - Ni)}{(99 \times 100)}$

$= \frac{(100\times 100\times Y - Ni)}{(99\times 100)}$

$$= \frac{(100 \times 100 \times Y - 100Y)}{(99 \times 100)}$$

$$= 99 \times 100 \times \frac{Y}{99} \times 100$$

$$= Y$$