Risk and Return (Part – 1) : Use of Geometric Mean and Arithmetic Mean

Topics to be Covered Are

- Meaning of Risk and Return
- Risk and Return of a Single Asset
- Risk and Return of a portfolio
- Capital Asset Pricing Model (CAPM)

Meaning of Risk and Return

- **Return**: Return is defined as the total gain or loss expected from a particular investment over a given period.
- **Risk**: Risk is defined as the variability of the return from the expected returns associated with a given investment/security.
- Risk and Return are two dimensions or determinants of the market price of a share.
- If the expected return to be generated from a particular investment project are more certain then the project is less risky.
- If the returns from an investment proposal increases, the risk attached to that proposal also increases.

To measure Risk and Return, we divide the discussion into two parts:

- Risk and Return of a single asset.
- Risk and Return of a portfolio.

Risk and Return of a Single Asset

Measuring Returns

Rate of Return: Measuring past return/expected return over a period of one year.

Return on any financial asset over a period of one year consists of the following:-

- Annual income received or expected to be received in the form of interest or dividend
- Capital gain or loss due to difference between the price in the beginning and at the end of the one year.

Hence, the rate of return on an asset for a given period is calculated as follows:

Rate of Return (R) = Dividend yield + Capital Gain yield

Or

$$= \frac{D1}{P0} X\ 100 + \frac{P1 - P0}{P0} X\ 100$$

Where, D_1 = Expected dividend per share at the end of the year one or return on investment

P_0 = Current Price or beginning price or initial investment

P_1 = Ending price or value of investment at the end of the year one

The return is calculated in percentage term as absolute return has no meaning.

Arithmetic Mean of Returns (Average Rate of Returns) : Measuring past return/expected return over a period of more than a year.

Arithmetic mean of return i.e.. , average of rate of return is defined as sum of returns of all observations divided by number of observation

$$\overline{R} = \frac{1}{n}(R1 + R2 + R3 + \ldots + \text{Rn}) \text{ or}$$

$$= \frac{1}{n} \sum_1^n \text{Rn}$$

Where, $\overline{R}$ = Arithmetic mean of return

R_1, R_2, R_3 ... R_n are the rate of return over period 1,2, 3, ... n respectively

n = No. of years in the holding period

Note: The arithmetic return ignores the compounding effect and order of returns and it is misleading when the investment returns are volatile.

Example 1: A Person invested ₹ 100 million in a project. The return for first 5 years was 5 % , 8 % , -2 % , 12 % and 9 % respectively. The arithmetic average return will equal 6.4 % i.e.. (5 %+ 8 %+ (-2 %) + 12 %+ 9 %) /5.

The investment value after 5 years will be ₹ 135.67 million as calculated below:

Investment value after 5 Years = ₹ 100 million × (1 + 5 %) × (1 + 8 %) × (1 – 2 %) × (1 + 12 %) × (1 + 9 %) = ₹ 135.67 million

However, the 6.4 % arithmetic average return suggest the investment value will be ₹ 145.09 million:

Investment value after 5 Years (based on Arithmetic Average Return) = ₹ 100 million × (1 + 6.4 %) ^{5} = ₹ 145.09 million

Arithmetic average return overstates the return because it ignores the order of return. For example, the decline of 2 % occurred in the investment when it had grown by 5 % and 8 % in the previous years, but arithmetic average return doesn't accommodate such compounding effect.

Geometric Mean of Returns: Geometric Mean gives one rate of return for the entire holding period of the asset. It takes compounding of returns into account.

$R_g = [(1 + R_1) (1 + R_2) (1 + R_3) \dots (1 + R_n)]^{1/n} - 1$

R_g = Geometric mean return

R_1, R_2, R_3 … R_n are the rate of return over period 1,2, 3, … n respectively

n = No. of years in the holding period

Use of Geometric Mean and Arithmetic Mean

Geometric Mean can be used to evaluate the past performance of an investment.

Arithmetic Mean of returns can be used to find the expected rate of return for future periods. There exists a relationship between arithmetic and geometric mean which is depicted in the following equation:-

$$R_g = R_a - \frac{1}{2}\sigma 2$$

Where, R_g = Geometric Mean

R_a = Arithmetic Mean

σ = Standard Deviation of returns

Internal Rate of Return (IRR)

IRR technique is defined as the rate of return which equates the present value of expected net cash inflows from investment proposal to the present value of its initial outflows.

Mathematically, IRR can be written as:

$$CO_o = \frac{CF\,1}{(1+k)\,1} + \frac{CF\,2}{(1+k)\,2} + \dots + \frac{CFn}{(1+k)\,n} + \frac{SV + WC}{(1+k)\,n}$$

where, CO_o = Cash outflow at time zero

CF_1, CF_2, CF_n, are the cash inflows at the end of year 1,2 ... n_{th}

SV = salvage value realized in n_{th} period (terminal year)

WC = working capital realized in n_{th} period (terminal year)

n = life span of the project in years

r = Internal rate of return (to be calculated)

- Hence at IRR: PV of net cash inflows = PV of net cash outflows
- If IRR so calculated is more than his desired rate of return, then investor can gain by making investment in the asset.
- Calculation of arithmetic and geometric mean of return does not consider the time value of money. Internal rate of return method of measuring return is based on cash flows and adopts discounting cash flow technique for adjusting time value of money.

Expected Rate of Return

The expected rate of return is the weighted average of all returns multiplied by their respective probabilities. Thus, probabilities of various outcomes are used as weights. The probability assigned to an outcome may vary between 0 and 1 and the sum of the probabilities assigned to various outcomes is 1. The formula for expected returns is given below.

$$- E(R) = \sum_{i=1}^{n} Pi\ Ri$$

where, E (R) = Expected Returns, P_i = Probability associated with R_i

R_i = Return for ith outcome, n = no. of outcomes considered

Measuring Risk

- Risk is defined as the variability of the actual return associated with the given investment/security. Higher the variability in the expected returns from an investment, higher would be the level of risk associated with that particular investment.
- Risk associated with single asset can be measured in many ways.

Some of the ways to measure risk associated with single asset are:

- Range
- Variance and Standard deviation of returns
- Coefficient of Variation

Range

- Range can be defined as the difference between the expected maximum and minimum values of return from the investment in a security. This is the simplest way of measuring the risk. Mathematically it can be calculated as follows:-
- Range = Maximum Value of return – Minimum value of return
- To understand, let us take an example of returns from the investment in two securities. A and B. Investment in both the securities is ₹ 100. The expected returns from the securities and calculation of range therefrom are given in Table.

Situation	A (Returns)	B(Returns)
Pessimistic	20	10
Most Likely	30	30
Optimistic	40	50
Range	(40 – 20 = 20)	(50 – 10 = 40)

From the data in Table, we can say that security B is riskier because the range between pessimistic and optimistic situation is more.

Variance and Standard Deviation of Returns

Variance: Variance is a measure of risk calculated by finding the sum of product of square of deviations of the individual return from the mean of expected returns and their respective probabilities. Following are the steps for calculating the variance.

- Determine the deviation of individual returns from the mean of expected returns.
- Find the square of deviations calculated in step (i) .
- Multiply the square of deviations with their respective probabilities.
- Find the sum of the product of square of deviations and their respective probabilities.

Mathematically it can be calculated as follows:

$$\text{Variance } (\sigma 2) = \sum_{i=1}^{n} [\text{Ri} - E(R)] 2X \text{ Pi}$$

Where, $\sigma 2$ = Variance, i = Outcome, R_i = Return for ith outcome

Pi = Probability associated with R_i

n = Number of outcomes considered, E (R) = Expected return

Standard Deviation: Standard Deviation is the most important and commonly used measure of risk. It can be calculated by finding the square root of the variance, i.e.. ,

$$\sigma = \sqrt{\text{Variance}} = \sqrt{\sum_{i=1}^{n} [\text{Ri} - E(R)]\, 2X\ \text{Pi}}$$

Where, σ = Standard Deviation

- The standard deviation is often used by investors to measure the risk of the investment. The basic idea is that the standard deviation is a measure of volatility the more an investment 's returns vary from the investment' s average return, the more volatile the investment.
- A Smaller Standard Deviation of an investment is less risky than the investment having higher Standard Deviation.

Coefficient of Variation: It is a measure of relative risk per unit of expected returns. It is calculated by dividing the standard deviation by its expected value.

$$\text{CV} = \frac{\sigma}{E(R)}$$

The rule is that larger the coefficient of variation, riskier will be the asset.

$$\left[\frac{D_1}{P_0} \times 100\right] + \left[\frac{P_1 - P_0}{P_0} \times 100\right]$$

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