

- (1) The end-points A and B of \overline{AB} are on the X- and Y-axis respectively. If $AB = a + b$, $a > 0$, $b > 0$, $a \neq b$ and P divides \overline{AB} from A in the ratio $b : a$, then show that P lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
- (2) If the feet of the perpendiculars drawn to the tangent at any point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from foci S and S' are L and L' respectively, then show that $SL \cdot S'L' = b^2$.
- (3) Prove that the line segment of any tangent, between the tangents at the end-points of the major axis, forms a right angle at either focus of the ellipse.
- (4) Show that the equation of the chord joining points P (α) and Q (β) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{x}{a} \cos \frac{\alpha + \beta}{2} + \frac{y}{b} \sin \frac{\alpha + \beta}{2} = \cos \frac{\alpha - \beta}{2}$.
- (5) If the chord joining the points P (α) and Q (β) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the focus (ae, 0), then prove that $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{e - 1}{e + 1}$.
- (6) If the chord joining the points P (α) and Q (β) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ subtends a right angle at the centre, then show that $\tan \alpha \cdot \tan \beta + \frac{a^2}{b^2} = 0$ and if it forms a right angle at the vertex (a, 0), then show that $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} + \frac{b^2}{a^2} = 0$.
- (7) If the difference of eccentric angles of the points P and Q on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{\pi}{2}$ and \overleftrightarrow{PQ} cuts intercepts of length c and d on the axes, then prove that $\frac{a^2}{c^2} + \frac{b^2}{d^2} = 2$.

(8) If two radii \overline{CP} and \overline{CQ} of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are perpendicular, then prove that $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} + \frac{1}{b^2}$, where C is the centre of the ellipse.

(9) Find the equations of the tangents drawn to the ellipse, $9x^2 + 16y^2 = 144$ from the point (2, 3).

[Ans: $x + y - 5 = 0$, $y - 3 = 0$]

(10) Find the equations of the tangents of the ellipse $9x^2 + 4y^2 = 36$ parallel to the line $y = 2x$. Also obtain the co-ordinates of the contact points.

[Ans: $2x - y + 5 = 0$ at $(-\frac{8}{5}, \frac{9}{5})$ and $2x - y - 5 = 0$ at $(\frac{8}{5}, -\frac{9}{5})$]

(11) Show that the tangents at the end-points of a focal chord of the ellipse intersect on the directrix.

(12) Show that the point of intersection of the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the points whose eccentric angles differ by $\frac{\pi}{2}$ lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

(13) The difference of the eccentric angles of the points P and Q on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{\pi}{2}$. If the tangents at P and Q intersect in R, then prove that \overline{CR} and \overline{PQ} bisect each other.

(14) P and Q are coherent points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its auxiliary circle. Show that the tangents to the ellipse at P and the circle at Q intersect on the X-axis. ($a > b$)

(15) If the lengths of the perpendicular line-segments from the centre to two mutually orthogonal tangents of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are p_1 and p_2 , then prove that $p_1^2 + p_2^2 = a^2 + b^2$.

(16) A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersects the axes in C and D respectively and touches the ellipse at mid-point of \overline{CD} in the first quadrant. Find its equation.

$$\left[\text{Ans: } \frac{x}{a} + \frac{y}{b} = \sqrt{2} \right]$$

(17) If the perpendicular distance of the focus S' from the tangent at a point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is p , then prove that $SP = \frac{2ap^2}{b^2 + p^2}$.

(18) $B(0, b)$ is one end-point of the chord of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) passing through the focus S' . If C is another end-point, then show that the slope of \overline{CP} is $\frac{(1 - e^2)^{3/2}}{2e}$, where C is the centre of the ellipse.

(19) The foot of the perpendicular from a point P on ellipse to the major axis is M . If \overline{PM} intersects the tangent at the end-point of a latus rectum in R , then prove that $\angle RSP = 90^\circ$.

(20) If the line containing a focal-chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersects the auxiliary circle in Q and Q' , then prove that $SQ \times SQ' = b^2$.

(21) Prove that if the tangent at a point P to the ellipse intersects a directrix at F , then \overline{PF} forms a right angle at the corresponding focus.

(22) The tangent at point P of an ellipse intersects the major axis in T. The line passing through T and perpendicular to major axis $\overline{AA'}$, intersects \overleftrightarrow{AP} and $\overleftrightarrow{A'P}$ in Q and Q' respectively. Show that T is the mid-point of $\overline{QQ'}$.

(23) The tangent at point P of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersects the axes in T and T' respectively. If R is the foot of perpendicular from the centre O to the tangent, then prove that $TT' \cdot PR = a^2 - b^2$.

(24) Find the condition that the line $lx + my + n = 0$ may be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and find the co-ordinates of its point of contact.

[Ans: $a^2l^2 + b^2m^2 = n^2$, $(-\frac{a^2l}{n}, -\frac{b^2m}{n})$]

(25) If the tangent at any point of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with centre C meets the major axis in T and minor axis in T', then prove that $\frac{a^2}{CT^2} + \frac{b^2}{CT'^2} = 1$ ($a > b$).

(26) Find the condition for the line $x \cos \alpha + y \sin \alpha = p$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

[Ans: $p \operatorname{cosec}^2 \alpha = a^2 \cot^2 \alpha + b^2$]

(27) P and Q are corresponding points on an ellipse and its auxiliary circle respectively. If the tangent at P to the ellipse meets the major axis in T, then show that \overleftrightarrow{QT} is a tangent to the auxiliary circle.

(28) Find the perpendicular distance between the tangents to the ellipse $\frac{x^2}{30} + \frac{y^2}{24} = 1$ which are parallel to the line $4x - 2y + 23 = 0$.

[Ans: $24 / \sqrt{5}$]

(29) Prove that the equation of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which has its mid-

point at (h, k) is $\frac{hx}{a^2} + \frac{ky}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$.

(30) Prove that the equation of the chord joining the points $P(\alpha + \beta)$ and $Q(\alpha - \beta)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{x}{a} \cos \alpha + \frac{y}{b} \sin \alpha = \cos \beta$.

(31) Prove that the area of the triangle formed by the points $P(\theta)$, $Q(\alpha)$ and $R(\beta)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $2ab \left| \sin \frac{\alpha - \theta}{2} \sin \frac{\beta - \theta}{2} \sin \frac{\theta - \alpha}{2} \right|$.

(32) Prove that the equations of the common tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = r^2$ ($b < r < a$) are $y\sqrt{a^2 - r^2} = \pm x\sqrt{r^2 - b^2} \pm r\sqrt{a^2 - b^2}$.

(33) Circles of constant radius c are drawn to pass through the ends of a variable diameter of the ellipse. Prove that the locus of their centres is the curve

$$(x^2 + y^2)(a^2 x^2 + b^2 y^2 + a^2 b^2) = c^2(a^2 x^2 + b^2 y^2).$$

(34) Prove that the measure of the angle between the two tangents drawn to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ from an external point } (h, k) \text{ is } \tan^{-1} \left| \frac{2ab \sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1}}{h^2 + k^2 - a^2 - b^2} \right|$$