

2. Electrostatic Potential and Capacitance

- Electrostatic potential at any point in a region of electrostatic field is the minimum work done in carrying a unit positive charge (without acceleration) from infinity to that point.
- Electrostatic forces are conservative.
- Electric potential due to a point charge:

$$V = W = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

The potential at a point with position vector r , due to a point dipole of dipole moment p placed at the origin is

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cdot \hat{r}}{r^2}$$

(The result is also true for a dipole with charges $-q$ and q separated by $2a$) for $r \gg a$.

- The electric potential at every point of an equipotential surface is the same. No work is done in moving the test charge from one point of the equipotential surface to the other.
- Relation between electric intensity and electric potential:

$$E = -\frac{dV}{dr}$$

- **Electrostatics of conductors**

- Inside a conductor, the electric field is zero.
- The interior of a conductor can have no excess charge in static situation.
- The electric field on the surface of a charged conductor is perpendicular to the surface of the conductor at every point.
- Electrostatic potential is constant throughout the volume of the conductor, and has the same value as on its surface.
- The electric field at the surface of a charged conductor is $\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$

Where, σ is the surface charge density and \hat{n} is a unit vector normal to the surface in the outward direction

- A capacitor is a system of two conductors separated by an insulator. Its capacitance is defined by

$$C = Q/V$$

Where, Q and $-Q$ are the charges on the two conductors and V is the potential difference between them

C is determined purely geometrically, by the shapes, sizes and relative positions of the two conductors. For a parallel plate capacitor (with vacuum between the plates),

$$C = \epsilon_0 \frac{A}{d}$$

Where, A is the area of each plate and d is the separation between them

- If the medium between the plates of a capacitor is filled with an insulating substance (dielectric), the electric field due to the charged plates induces a net dipole moment in the dielectric. This effect, called polarisation, gives rise to a field in the opposite direction. The net electric field inside the dielectric, and hence the potential difference between the plates, is thus reduced.

Consequently, the capacitance C increases from its value C_0 {when there is no medium (vacuum)}, $C = KC_0$

Where, K is the dielectric constant of the insulating substance

- For capacitors in series combination, the total capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

In parallel combination, the total capacitance C is

$$C = C_1 + C_2 + C_3 + \dots$$

- The Energy U stored in a capacitor of capacitance C , with charge Q and voltage V is

$$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

