

Descriptive Statistics:

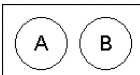
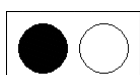


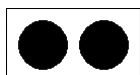
Term	Meaning	Population Formula	Sample Formula	Example {1,16,1,3,9}
Sort	Sort values in increasing order			{1,1,3,9,16}
Mean	Average	$\mu = \frac{\sum_{i=1}^N X_i}{N}$	$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n} = \frac{\sum_{i=1}^n X_i}{n}$	6
Median	The middle value – half are below and half are above			3
Mode	The value with the most appearances			1
Variance	The average of the squared deviations between the values and the mean	$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (X_i - \mu)^2$	$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}$	$(1-6)^2 + (1-6)^2 + (3-6)^2 + (9-6)^2 + (16-6)^2$ divided by 5 values = 168/5 = 33.6
Standard Deviation	The square root of Variance, thought of as the “average” deviation from the mean.	$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$	Square root of 33.6 = 5.7966
Coefficient of Variation	The variation relative to the value of the mean		$CV = \frac{s}{\bar{X}}$	5.7966 divided by 6 = 0.9661
Minimum	The minimum value			1
Maximum	The maximum value			16
Range	Maximum minus Minimum			16 – 1 = 15

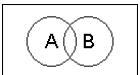
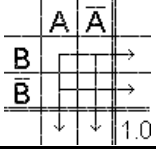
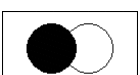
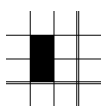

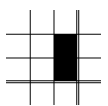
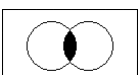
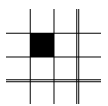

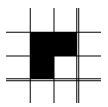
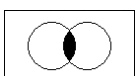
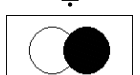
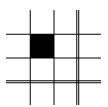
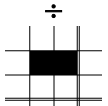
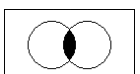
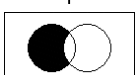
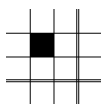
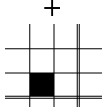
Probability Terms:

Term	Meaning	Notation	Example* (see footnote)
Probability	For any event A, probability is represented within $0 \leq P \leq 1$.	P()	0.5
Random Experiment	A process leading to at least 2 possible outcomes with uncertainty as to which will occur.		Rolling a dice
Event	A subset of all possible outcomes of an experiment.		Events A and B
Intersection of Events	Let A and B be two events. Then the intersection of the two events is the event that both A and B occur (logical AND).	$A \cap B$	The event that a 2 appears
Union of Events	The union of the two events is the event that A or B (or both) occurs (logical OR).	$A \cup B$	The event that a 1, 2, 4, 5 or 6 appears
Complement	Let A be an event. The complement of A is the event that A does not occur (logical NOT).	\bar{A}	The event that an odd number appears
Mutually Exclusive Events	A and B are said to be mutually exclusive if at most one of the events A and B can occur.		A and B are not mutually exclusive because if a 2 appears, both A and B occur
Collectively Exhaustive Events	A and B are said to be collectively exhaustive if at least one of the events A or B must occur.		A and B are not collectively exhaustive because if a 3 appears, neither A nor B occur
Basic Outcomes	The simple indecomposable possible results of an experiment. One and exactly one of these outcomes must occur. The set of basic outcomes is mutually exclusive and collectively exhaustive.		Basic outcomes 1, 2, 3, 4, 5, and 6
Sample Space	The totality of basic outcomes of an experiment.		{1,2,3,4,5,6}

* Roll a fair die once. Let A be the event an even number appears, let B be the event a 1, 2 or 5 appears

Probability Rules:

If events A and B are <u>mutually exclusive</u>		
<u>Term</u>	<u>Equals</u>	<u>Area:</u> 
$P(A)=$	$P(A)$	
$P(\bar{A})=$	$1 - P(A)$	
$P(A \cap B)=$	0	
$P(A \cup B)=$	$P(A) + P(B)$	

If events A and B are <u>NOT mutually exclusive</u>			
<u>Term</u>	<u>Equals</u>	<u>Venn:</u> 	
$P(A)=$	$P(A)$		
$P(\bar{A})=$	$1 - P(A)$		
$P(A \cap B)=$	$P(A) * P(B)$ <i>only if A and B are independent</i>		
$P(A \cup B)=$	$P(A) + P(B) - P(A \cap B)$		
$P(A B)=$	$\frac{P(A \cap B)}{P(B)}$ /Bayes' Law: <i>P(A holds given that B holds)</i>	 \div 	 \div 
$P(A \cap B) = P(A B) * P(B)$			
$P(A \cap B) = P(B A) * P(A)$			
$P(A)=$	$P(A \cap B) + P(A \cap \bar{B}) = P(A B)P(B) + P(A \bar{B})P(\bar{B})$	 $+$ 	 $+$ 
*Example: Take a deck of 52 cards. Take out 2 cards sequentially, but don't look at the first. The probability that the second card you chose was a ♣ is the probability of choosing a ♣ (event A) after choosing a ♣ (event B), plus the probability of choosing a ♣ (event A) after not choosing a ♣ (event B), which equals $(12/51)(13/52) + (13/51)(39/52) = 1/4 = 0.25$.			

General probability rules:

1) If $P(A|B) = P(A)$, then A and B are **independent events!** (for example, rolling dice one after the other).

2) If there are n possible outcomes which are equally likely to occur:

$$P(\text{outcome } i \text{ occurs}) = \frac{1}{n} \text{ for each } i \in [1, 2, \dots, n]$$

**Example: Shuffle a deck of cards, and pick one at random. $P(\text{chosen card is a } 10 \spadesuit) = 1/52$.*

3) If event A is composed of n equally likely basic outcomes:

$$P(A) = \frac{\text{Number of Basic Outcomes in } A}{n}$$

**Example: Suppose we toss two dice. Let A denote the event that the sum of the two dice is 9. $P(A) = 4/36 = 1/9$, because there are 4 out of 36 basic outcomes that will sum 9.*

Random Variables and Distributions:

To calculate the **Expected Value** $E(X) = \sum x \cdot P(X = x)$, use the following table:

Event	Payoff	Probability	Weighted Payoff
[name of first event]	[payoff of first event in \$]	[probability of first event $0 \leq P \leq 1$]	[product of Payoff * Probability]
[name of second event]	[payoff of second event in \$]	[probability of second event $0 \leq P \leq 1$]	[product of Payoff * Probability]
[name of third event]	[payoff of third event in \$]	[probability of third event $0 \leq P \leq 1$]	[product of Payoff * Probability]
Total (Expected Payoff):			[total of all Weighted Payoffs above]

* See example in BOOK 1 page 54

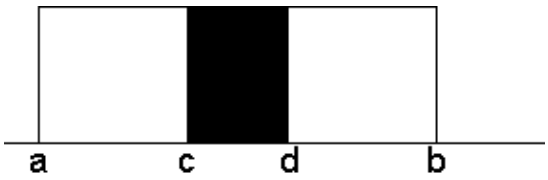
To calculate the **Variance** $\text{Var}(X) = \sum (x - E(X))^2 P(X = x)$ and **Standard Deviation** $\sigma(X) = \sqrt{\text{Var}(X)}$, use:

Event	Payoff	Expected Payoff	Error	(Error) ²	Probability	Weighted (Error) ²
[1 st event]	[1 st payoff]	[Total from above]	[1 st payoff minus Expected Payoff]	1 st Error squared	1 st event's probability	1 st (Error) ² * 1 st event's probability
[2 nd event]	[2 nd payoff]	[Total from above]	[2 nd payoff minus Expected Payoff]	2 nd Error squared	2 nd event's probability	2 nd (Error) ² * 2 nd event's probability
[3 rd event]	[3 rd payoff]	[Total from above]	[3 rd payoff minus Expected Payoff]	3 rd Error squared	3 rd event's probability	3 rd (Error) ² * 3 rd event's probability
Variance:						[total of above]
Std. Deviation:						[square root of Variance]

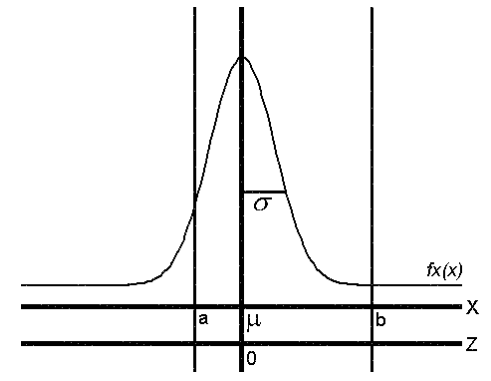
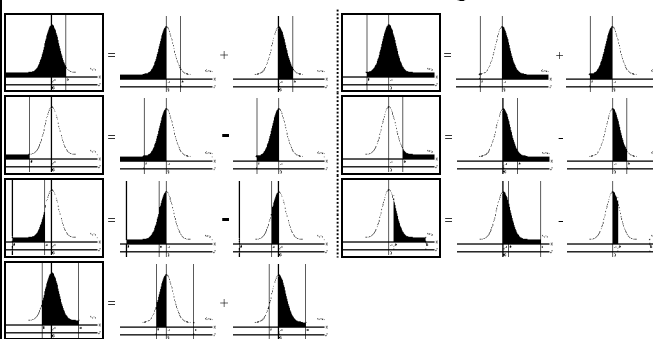
Counting Rules:

Term	Meaning	Formula	Example
Basic Counting Rule	The <i>number</i> of ways to pick x things out of a set of n (with no regard to order). The <i>probability</i> is calculated as $1/x$ of the result.	$\binom{n}{x} = \frac{n!}{x!(n-x)!}$	The <i>number</i> of ways to pick 4 specific cards out of a deck of 52 is: $52!/((4!)(48!)) = 270,725$, and the <i>probability</i> is $1/270,725 = 0.000003694$
Bernoulli Process	For a sequence of n trials, each with an outcome of either success or failure, each with a probability of p to succeed – the probability to get x successes is equal to the Basic Counting Rule formula (above) times $p^x(1-p)^{n-x}$.	$P(X = x n, p) = \left(\frac{n!}{x!(n-x)!} \right) p^x (1-p)^{n-x}$	If an airline takes 20 reservations, and there is a 0.9 probability that each passenger will show up, then the probability that exactly 16 passengers will show is: $\frac{20!}{16!4!} (0.9)^{16}(0.1)^4 = 0.08978$
Bernoulli Expected Value	The expected value of a Bernoulli Process, given n trials and p probability.	$E(X) = np$	In the example above, the number of people expected to show is: $(20)(0.9) = 18$
Bernoulli Variance	The variance of a Bernoulli Process, given n trials and p probability.	$\text{Var}(X) = np(1-p)$	In the example above, the Bernoulli Variance is $(20)(0.9)(0.1) = 1.8$
Bernoulli Standard Deviation	The standard deviation of a Bernoulli Process:	$\sigma(X) = \sqrt{np(1-p)}$	In the example above, the Bernoulli Standard Deviation is $\sqrt{1.8} = 1.34$
Linear Transformation Rule	If X is random and $Y = aX + b$, then the following formulas apply:	$\begin{aligned} E(Y) &= a \cdot E(X) + b \\ \text{Var}(Y) &= a^2 \cdot \text{Var}(X) \\ \sigma(Y) &= a \cdot \sigma(X) \end{aligned}$	

Uniform Distribution:

	Term/Meaning	Formula
	Expected Value	$\bar{X} = \frac{(a+b)}{2}$
	Variance	$\sigma_x^2 = \frac{(b-a)^2}{12}$
	Standard Deviation	$\sigma_x = \frac{(b-a)}{\sqrt{12}}$
	Probability that X falls between c and d	$P(c \leq X \leq d) = \frac{d-c}{b-a}$

Normal Distribution:

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<td>2.3</td> <td>.4893</td><td>.4896</td><td>.4898</td><td>.4901</td><td>.4904</td><td>.4906</td><td>.4909</td><td>.4911</td><td>.4913</td><td>.4916</td> </tr> <tr> <td>2.4</td> <td>.4918</td><td>.4920</td><td>.4922</td><td>.4925</td><td>.4927</td><td>.4929</td><td>.4931</td><td>.4932</td><td>.4934</td><td>.4936</td> </tr> <tr> <td>2.5</td> <td>.4938</td><td>.4940</td><td>.4941</td><td>.4943</td><td>.4945</td><td>.4946</td><td>.4948</td><td>.4949</td><td>.4951</td><td>.4952</td> </tr> <tr> <td>2.6</td> <td>.4953</td><td>.4955</td><td>.4956</td><td>.4957</td><td>.4959</td><td>.4960</td><td>.4961</td><td>.4962</td><td>.4963</td><td>.4964</td> </tr> <tr> <td>2.7</td> <td>.4965</td><td>.4966</td><td>.4967</td><td>.4968</td><td>.4969</td><td>.4970</td><td>.4971</td><td>.4972</td><td>.4973</td><td>.4974</td> </tr> <tr> <td>2.8</td> <td>.4974</td><td>.4975</td><td>.4976</td><td>.4977</td><td>.4977</td><td>.4978</td><td>.4979</td><td>.4979</td><td>.4980</td><td>.4981</td> </tr> <tr> <td>2.9</td> <td>.4981</td><td>.4982</td><td>.4982</td><td>.4983</td><td>.4984</td><td>.4984</td><td>.4985</td><td>.4985</td><td>.4986</td><td>.4986</td> </tr> <tr> <td>3.0</td> <td>.4987</td><td>.4987</td><td>.4987</td><td>.4988</td><td>.4988</td><td>.4989</td><td>.4989</td><td>.4989</td><td>.4990</td><td>.4990</td> </tr> </table>	z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753	0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549	0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133	0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981	2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09																																																																																																																																																																																																																																																																																																																																																							
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359																																																																																																																																																																																																																																																																																																																																																							
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753																																																																																																																																																																																																																																																																																																																																																							
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141																																																																																																																																																																																																																																																																																																																																																							
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517																																																																																																																																																																																																																																																																																																																																																							
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879																																																																																																																																																																																																																																																																																																																																																							
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224																																																																																																																																																																																																																																																																																																																																																							
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549																																																																																																																																																																																																																																																																																																																																																							
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852																																																																																																																																																																																																																																																																																																																																																							
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133																																																																																																																																																																																																																																																																																																																																																							
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389																																																																																																																																																																																																																																																																																																																																																							
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621																																																																																																																																																																																																																																																																																																																																																							
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830																																																																																																																																																																																																																																																																																																																																																							
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015																																																																																																																																																																																																																																																																																																																																																							
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177																																																																																																																																																																																																																																																																																																																																																							
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319																																																																																																																																																																																																																																																																																																																																																							
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441																																																																																																																																																																																																																																																																																																																																																							
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545																																																																																																																																																																																																																																																																																																																																																							
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633																																																																																																																																																																																																																																																																																																																																																							
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706																																																																																																																																																																																																																																																																																																																																																							
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767																																																																																																																																																																																																																																																																																																																																																							
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817																																																																																																																																																																																																																																																																																																																																																							
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857																																																																																																																																																																																																																																																																																																																																																							
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890																																																																																																																																																																																																																																																																																																																																																							
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916																																																																																																																																																																																																																																																																																																																																																							
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936																																																																																																																																																																																																																																																																																																																																																							
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952																																																																																																																																																																																																																																																																																																																																																							
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964																																																																																																																																																																																																																																																																																																																																																							
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974																																																																																																																																																																																																																																																																																																																																																							
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981																																																																																																																																																																																																																																																																																																																																																							
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986																																																																																																																																																																																																																																																																																																																																																							
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990																																																																																																																																																																																																																																																																																																																																																							
<p>Probability Density Function:</p> $f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ <p>where $\pi \approx 3.1416$ and $e \approx 2.7183$</p>																																																																																																																																																																																																																																																																																																																																																																	
<p>Standard Deviations away from the mean:</p> $Z = \frac{X - \mu}{\sigma}$ <p>(Z and σ are swappable!)</p>																																																																																																																																																																																																																																																																																																																																																																	
<p>$P(a \leq X \leq b) = \text{area under } f_x(x) \text{ between } a \text{ and } b:$</p> $P(a \leq X \leq b) = P\left[\left(\frac{a - \mu}{\sigma}\right) \leq Z \leq \left(\frac{b - \mu}{\sigma}\right)\right]$																																																																																																																																																																																																																																																																																																																																																																	
<p>Standard Normal Table - seven usage scenarios:</p> 																																																																																																																																																																																																																																																																																																																																																																	

Correlation:

➤ If X and Y are two different sets of data, their correlation is represented by $\text{Corr}_{(XY)}$, r_{XY} , or ρ_{XY} (rho).

- If Y increases as X increases, $0 < \rho_{XY} < 1$. If Y decreases as X increases, $-1 < \rho_{XY} < 0$.
- The extremes $\rho_{XY} = 1$ and $\rho_{XY} = -1$ indicated perfect correlation – info about one results in an exact prediction about the other.
- If X and Y are completely uncorrelated, $\rho_{XY} = 0$.
- The **Covariance** of X and Y, $Cov_{(XY)}$, has the same sign as ρ_{XY} , has unusual units and is usually a means to find ρ_{XY} .

Term	Formula	Notes
Correlation	$Corr_{(XY)} = \frac{Cov_{(XY)}}{\sigma_X \sigma_Y}$	Used with Covariance formulas below
Covariance (2 formulas)	$Cov_{(XY)} = E[(X - \bar{X})(Y - \bar{Y})]$ <i>(difficult to calculate)</i>	Sum of the products of all sample pairs' distance from their respective means multiplied by their respective probabilities
	$Cov_{(XY)} = E(XY) - (\bar{X})(\bar{Y})$	Sum of the products of all sample pairs multiplied by their respective probabilities, minus the product of both means
Finding Covariance given Correlation	$Cov_{(XY)} = \sigma_X \sigma_Y Corr_{(XY)}$	

Portfolio Analysis:

	Term	Formula	Example*
	Mean of any Portfolio "S"	$\bar{S} = a\bar{X} + b\bar{Y}$	$\bar{S} = \frac{3}{4}(8.0\%) + \frac{1}{4}(11.0\%) = 8.75\%$
<i>Uncorrelated</i>	Portfolio Variance	$\sigma^2 = a^2 \sigma_X^2 + b^2 \sigma_Y^2$	$\sigma^2 = (\frac{3}{4})^2(0.5)^2 + (\frac{1}{4})^2(6.0)^2 = 2.3906$
	Portfolio Standard Deviation	$\sigma = \sqrt{a^2 \sigma_X^2 + b^2 \sigma_Y^2}$	$\sigma = 1.5462$
<i>Correlated</i>	Portfolio Variance	$\sigma_{(aX+bY)}^2 = a^2 \sigma_X^2 + b^2 \sigma_Y^2 + 2abCov_{(XY)}$	
	Portfolio Standard Deviation	$\sigma_{(aX+bY)} = \sqrt{a^2 \sigma_X^2 + b^2 \sigma_Y^2 + 2abCov_{(XY)}}$	

* Portfolio "S" composed of $\frac{3}{4}$ Stock A (mean return: 8.0%, standard deviation: 0.5%) and $\frac{1}{4}$ Stock B (11.0%, 6.0% respectively)

The Central Limit Theorem

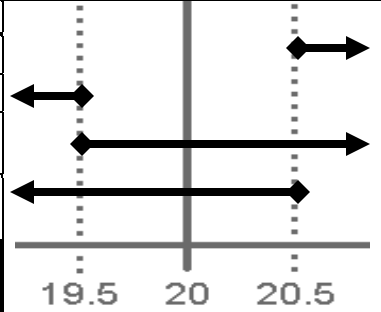
Normal distribution can be used to approximate binominals of more than 30 trials ($n \geq 30$):

Term	Formula
Mean	$E(X) = np$
Variance	$Var(X) = np(1 - p)$
Standard Deviation	$\sigma(X) = \sqrt{np(1 - p)}$

Continuity Correction

Unlike continuous (normal) distributions (i.e. \$, time), discrete binomial distribution of integers (i.e. # people) must be corrected:

Old cutoff	New cutoff
$P(X > 20)$	$P(X > 20.5)$
$P(X < 20)$	$P(X < 19.5)$
$P(X \geq 20)$	$P(X \geq 19.5)$
$P(X \leq 20)$	$P(X \leq 20.5)$



Sampling Distribution of the Mean

If the X_i 's are normally distributed (or $n \geq 30$), then \bar{X} is normally distributed with:

Term	Formula
Mean	μ
Standard Error of the Mean	$\frac{\sigma}{\sqrt{n}}$

Sampling Distribution of a Proportion

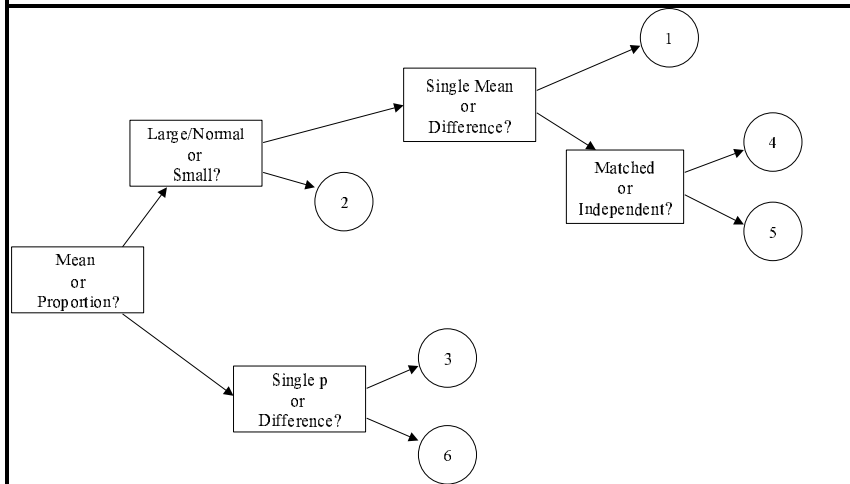
If, for a proportion, $n \geq 30$ then \hat{p} is normally distributed with:

Term	Formula
Mean	p
Standard Deviation	$\sqrt{\frac{p(1 - p)}{n}}$

Confidence Intervals:

	Parameter	Confidence Interval	Usage	Sample	σ
1	μ	$\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$	Normal		Known σ
	μ	$\bar{X} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$		Large	Unknown σ
2	μ	$\bar{X} \pm t_{(n-1, \alpha/2)} \frac{s}{\sqrt{n}}$	Normal	Small	Unknown σ
3	p	$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	Binomial	Large	
4	$\mu_X - \mu_Y$	$\bar{D} \pm z_{\alpha/2} \frac{s_D}{\sqrt{n}}$	Normal		Matched pairs
5	$\mu_X - \mu_Y$	$\bar{X} - \bar{Y} \pm z_{(\alpha/2)} \sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}$	Normal		Known σ , Independent Samples
	$\mu_X - \mu_Y$	$\bar{X} - \bar{Y} \pm z_{(\alpha/2)} \sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}$		Large	
6	$p_X - p_Y$	$\hat{p}_X - \hat{p}_Y \pm z_{(\alpha/2)} \sqrt{\frac{\hat{p}_X(1-\hat{p}_X)}{n_X} + \frac{\hat{p}_Y(1-\hat{p}_Y)}{n_Y}}$	Binomial	Large	

Formulae Guide



Confidence Level to Z-Value Guide

Confidence Level		$Z_{\alpha/2}$ (2-Tail)	Z_{α} (1-Tail)
80%	$\alpha = 20\%$	1.28	0.84
90%	$\alpha = 10\%$	1.645	1.28
95%	$\alpha = 5\%$	1.96	1.645
99%	$\alpha = 1\%$	2.575	2.325
c	$\alpha = 1.0-c$	$Z_{(c/2)}$	$Z_{(c-0.5)}$

Determining the Appropriate Sample Size

Term	Normal Distribution Formula	Proportion Formula
Sample Size (for +/- e)	$n = \frac{(1.96)^2 \sigma^2}{e^2}$	$n \geq \frac{1.96^2}{4e^2}$

t-table

d. f.	0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.656
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763

Hypothesis Testing:

Test Type	Test Statistic	Two-tailed		Lower-tail		Upper-tail	
		H _a	Critical Value	H _a	Critical Value	H _a	Critical Value
Single μ ($n \geq 30$)	$z_0 = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$	$\mu \neq \mu_0$	$\pm z_{\alpha/2}$	$\mu < \mu_0$	$-z_{\alpha}$	$\mu > \mu_0$	$+z_{\alpha}$
Single μ ($n < 30$)	$t_0 = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$	$\mu \neq \mu_0$	$\pm t_{(n-1, \alpha/2)}$	$\mu < \mu_0$	$-t_{(n-1, \alpha)}$	$\mu > \mu_0$	$+t_{(n-1, \alpha)}$
Single p ($n \geq 30$)	$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$p \neq p_0$	$\pm z_{\alpha/2}$	$p < p_0$	$-z_{\alpha}$	$p > p_0$	$+z_{\alpha}$
Diff. between two μ s	$z_0 = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}}$	$\mu_X - \mu_Y \neq 0$	$\pm z_{\alpha/2}$	$\mu_X - \mu_Y < 0$	$-z_{\alpha}$	$\mu_X - \mu_Y > 0$	$+z_{\alpha}$
Diff. between two p s	$z_0 = \frac{(\hat{p}_X - \hat{p}_Y) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{n_X + n_Y}{n_X n_Y}\right)}}$	$p_X - p_Y \neq 0$	$\pm z_{\alpha/2}$	$p_X - p_Y < 0$	$-z_{\alpha}$	$p_X - p_Y > 0$	$+z_{\alpha}$

Classic Hypothesis Testing Procedure		
Step	Description	Example
1	Formulate Two Hypotheses	The hypotheses ought to be mutually exclusive and collectively exhaustive. The hypothesis to be tested (the null hypothesis) always contains an equals sign, referring to some proposed value of a population parameter. The alternative hypothesis never contains an equals sign, but can be either a one-sided or two-sided inequality. $H_0: \mu = 0$ $H_A: \mu < 0$
2	Select a Test Statistic	The test statistic is a standardized estimate of the difference between our sample and some hypothesized population parameter. It answers the question: "If the null hypothesis were true, how many standard deviations is our sample away from where we expected it to be?" $\frac{\bar{X} - \mu_0}{s/\sqrt{n}}$
3	Derive a Decision Rule	The decision rule consists of regions of rejection and non-rejection, defined by critical values of the test statistic. It is used to establish the probable truth or falsity of the null hypothesis. We reject H_0 if $\bar{X} < \mu_0 - z_{\alpha} \frac{\sigma}{\sqrt{n}}$
4	Calculate the Value of the Test Statistic; Invoke the Decision Rule in light of the Test Statistic	Either reject the null hypothesis (if the test statistic falls into the rejection region) or do not reject the null hypothesis (if the test statistic does not fall into the rejection region). $\frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{-0.21 - 0}{0.80/\sqrt{50}}$

Regression:

Statistic	Symbol
Independent Variables	X_1, \dots, X_k
Dependent Variable (a random variable)	Y
Dependent Variable (an individual observation among sample)	Y_i
Intercept (or constant); an unknown population parameter	β_0
Estimated intercept; an estimate of β_0	$\hat{\beta}_0$
Slope (or coefficient) for Independent Variable 1 (unknown)	β_1
Estimated slope for Independent Variable 1; an estimate of β_1	$\hat{\beta}_1$

Regression Statistics					
Multiple R	0.9568				
R Square	0.9155				
Adjusted R Square	0.9015				
Standard Error	6.6220				
Observations	15				
ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	2	5704.0273	2852.0137	65.0391	0.0000
Residual	12	526.2087	43.8507		
Total	14	6230.2360			
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	
Intercept	-20.3722	9.8139	-2.0758	0.0601	
Size (100 sq ft)	4.3117	0.4104	10.5059	0.0000	
Lot Size (1000 sq ft)	4.7177	0.7646	6.1705	0.0000	

Statistic (Mapped to Output Above)	Symbol	Formula
Dependent Variable (sample mean of n observations)	\bar{Y}	$\bar{Y} = \frac{\sum_{i=1}^n Y_i}{n}$
Dependent Variable (estimated value for a given vector of independent variables)	\hat{Y}_i	$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{1i} + \hat{\beta}_2 x_{2i} + \hat{\beta}_3 x_{3i} + \dots + \hat{\beta}_k x_{ki}$
Error for observation i. The unexplained difference between the actual value of Y_i and the prediction for Y_i based on our regression model.	ε_i	$\varepsilon_i = Y_i - \hat{Y}_i$
6230.2360 Total Sum of Squares	TSS (or SST)	$TSS = \sum_{i=1}^n (Y_i - \bar{Y})^2 = SSR + SSE$
526.2087 Sum of Squares due to Error	SSE	$SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$
43.8507 Mean Squares due to Error	MSE	$MSE = \frac{SSE}{n - k - 1}$
5704.0273 Sum of Squares due to Regression	SSR	$SSR = \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2$
2852.0137 Mean Squares due to Regression	MSR	$MSR = \frac{SSR}{k}$

Statistic (Mapped to Output Above)	Symbol	Formula
0.9155 R-square (Coefficient of Determination)	R^2	$R^2 = 1 - \frac{SSE}{TSS}$
0.9568 Multiple R (Coefficient of Multiple Correlation)	R	$R = \sqrt{R^2}$
0.9015 Adjusted R-square	\bar{R}^2	$\bar{R}^2 = 1 - \frac{\left(\frac{SSE}{n - k - 1}\right)}{\left(\frac{SST}{n - 1}\right)}$
6.6220 Standard Error (a.k.a. Standard Error of the Estimate)	s_ε	$s_\varepsilon = \sqrt{\frac{SSE}{n - k - 1}}$
-2.0758 t-statistic for testing $H_0 : \beta_1 = 0$ vs. $H_A : \beta_1 \neq 0$	t_0	$t_0 = \frac{\hat{\beta}_1 - 0}{s_{\beta_1}}$
0.0601 p-value for testing $H_0 : \beta_1 = 0$ vs. $H_A : \beta_1 \neq 0$	p-value	$P(T > t_0)$
65.0391 F	F	$F = \frac{MSR}{MSE}$ $= \frac{n - k - 1}{k} \times \frac{R^2}{1 - R^2}$