10.1 Laws of reflection (applicable to both the plane as well as the curved surfaces)

(1) The angle of incidence is equal to the angle of reflection.

(2) Incident ray, reflected ray and the normal drawn at the point of incidence are in the same plane.

10.2 Reflection of Light by Spherical Mirrors

Concave mirror is formed by making the inner surface of the circular cross-section of a spherical shell reflecting while the convex mirror is formed by making the outer surface reflecting.

Some definitions with reference to the mirror (Refer to the figures as under.)

(1) Pole (P) - centre of the reflecting surface

(2) Principal Axis - the imaginary line passing through the pole and centre of curvature of the mirror

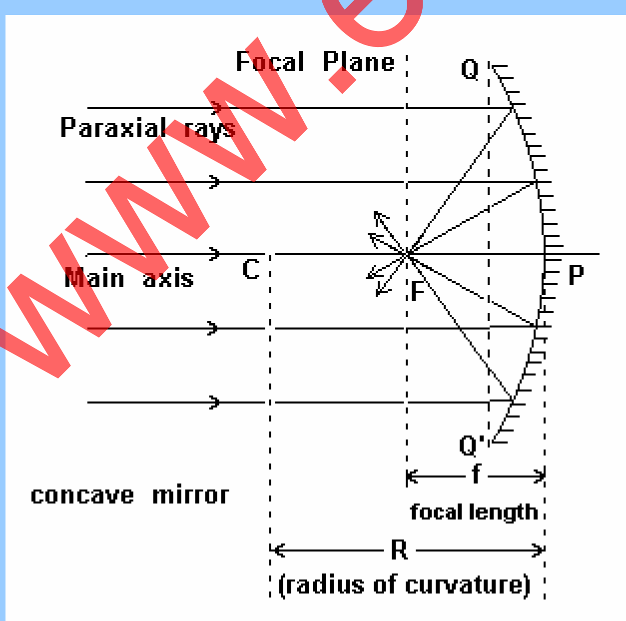
(3) Aperture (QQ') - diameter of the reflecting surface

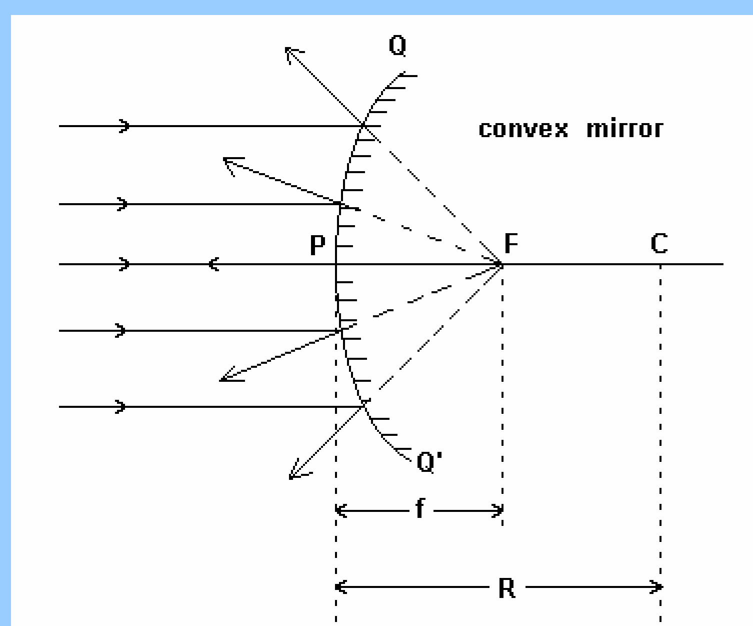
(4) Principal Focus - the point where the ray parallel to the principal axis meet (concave mirror) or appear to meet (convex mirror), after reflection

(5) Focal Plane - plane passing through the principal focus and normal to the principal axis

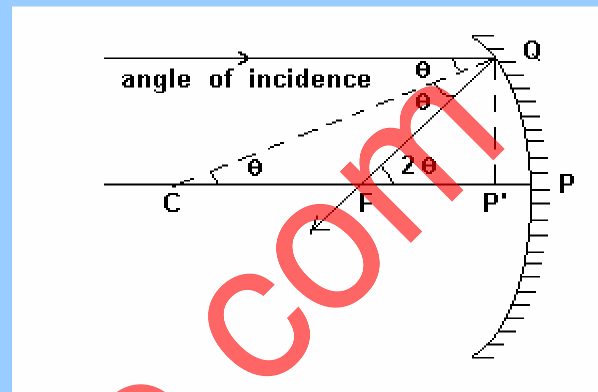
(6) Focal Length - the distance between the pole and the principal focus

(7) Paraxial Rays - rays close to the principal axis





10.3 Relation Between Focal Length and Radius of Curvature



As shown in the figure, a paraxial ray is incident at point Q on a concave mirror.

θ = angle of incidence = angle of reflection
= $\angle$CQF = $\angle$QCF (by geometry)

So, for Δ CFQ,
exterior $\angle$ QFP = $\angle$ CQF + $\angle$ QCF = 2θ .

For paraxial incident ray and small aperture,
CP' $\approx$ CP = R and FP' $\approx$ FP = f.

For small aperture, 2θ is very small.

$\therefore$ from the figure, $2\theta \approx \dfrac{QP}{FP} = \dfrac{QP}{f}$ ... (1) and $\theta = \dfrac{QP}{CP} = \dfrac{QP}{R}$ ... (2)

From equations (1) and (2), R = 2f $\Rightarrow$ f = R / 2

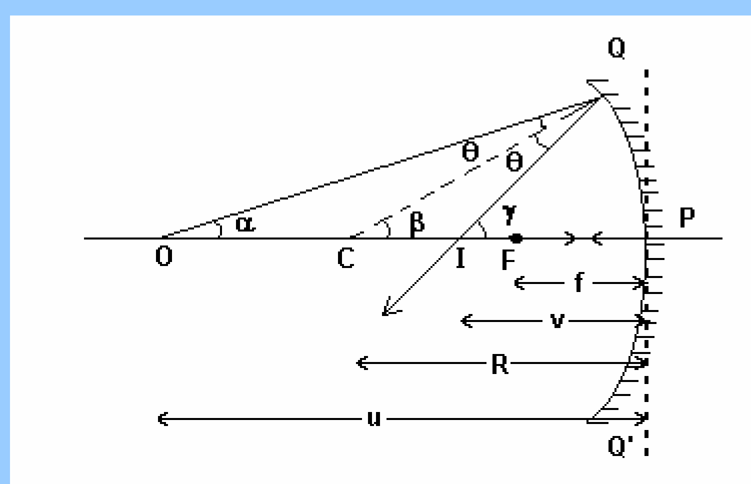
Thus, focal length of a concave mirror is half its radius of curvature.

Sign Convention

Sign convention for the object distance (u), image distance (v), focal length (f) and radius of curvature (R) in the formulae to be derived are as under.

(1) All distances are measured on the principal axis from the pole of the mirror.
(2) Distance in the direction of incident ray is positive and opposite to it negative.
(3) Height above the principal axis is positive and below it is negative.

Mirror Formula



As shown in the figure, a ray from object O, at a distance u, is incident at point Q on the concave mirror of small aperture making an angle α with the principal axis. It gets reflected in the direction QI making the same angle θ with the normal CQ as the incident ray.

Another ray from O, moving along the axis, is incident at point P and gets reflected in the direction PC.

Both these rays meet at I on the principal axis forming image of the object at a distance v from the pole.

According to the laws of reflection,
angle of incidence, $\angle$OQC = angle of reflection, $\angle$CQI = θ

CQ and IQ make angles β and γ respectively with the principal axis.

In Δ OCQ, exterior angle $\beta = \alpha + \theta$

In Δ CQI, exterior angle $\gamma = \beta + \theta$ Eliminating θ, $\alpha + \gamma = 2\beta$

Now, $\alpha \text{ (rad)} \approx \dfrac{\text{arc QP}}{\text{OP}}$, $\beta = \dfrac{\text{arc QP}}{\text{CP}}$ and $\gamma \approx \dfrac{\text{arc QP}}{\text{IP}}$

Putting these values in the above equation,

$$\frac{\text{arc QP}}{\text{OP}} + \frac{\text{arc QP}}{\text{IP}} = 2\frac{\text{arc QP}}{\text{CP}} \qquad \therefore \qquad \frac{1}{\text{OP}} + \frac{1}{\text{IP}} = \frac{2}{\text{CP}}$$

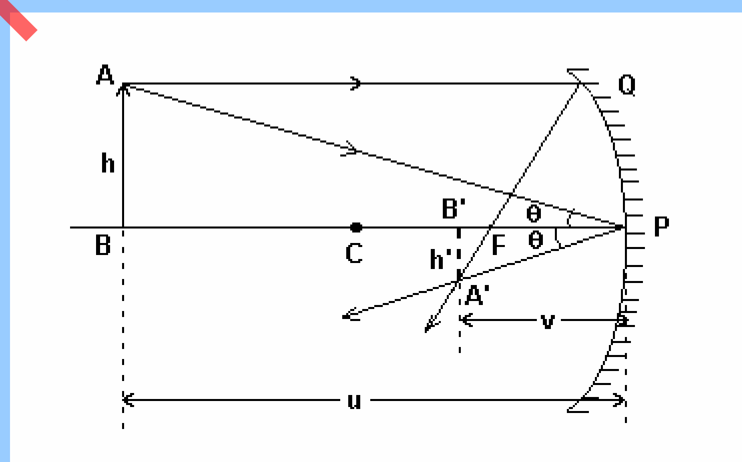
But as all distances are in direction opposite to the incident ray,

OP = - u, CP = - R and IP = - v,

$$\therefore \quad \frac{1}{-u} + \frac{1}{-v} = \frac{2}{-R} \qquad\qquad \therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

This is called Gauss' equation for a curved mirror. It is also valid for a convex mirror.

Magnification due to a Mirror



AB is the object at a distance u on the axis of a concave mirror as shown in the figure.

A ray AQ, parallel to the principal axis and ray AP, incident on the pole, meet at point A' after reflection and form the image of A. A'B' normal to the axis is the image of AB.

The ratio of the height of the image to the height of the object is called the transverse magnification or lateral magnification.

$$\therefore \text{ lateral magnification, } m = \frac{\text{height of the image}}{\text{height of the object}} = \frac{h'}{h}$$

Δs ABP and A'B'P are similar. $\therefore \dfrac{A'B'}{AB} = \dfrac{B'P}{BP}$

Hence, using proper sign convention, $\dfrac{-h'}{h} = \dfrac{-v}{-u} = \dfrac{v}{u}$

$$\therefore \quad \text{Lateral magnification} = \frac{-h'}{h} = \frac{v}{u}$$

The same equation is obtained for a convex mirror.

10.4 Refraction of Light

When a ray of light goes from one transparent medium to another, its direction changes at the boundary surface (unless it is incident normally to the surface). This phenomenon is called refraction.

Laws of Refraction

(1) The incident ray, refracted ray and the normal drawn to the point of incidence are in the same plane.

(2) "For the two given media if θ_1 is the angle of incidence and θ_2 is the angle of refraction, then the ratio $\frac{\sin\theta_1}{\sin\theta_2}$ is a constant." (Snell's law)

This ratio, n_{21}, is called the refractive index of medium (2) with respect to medium (1).

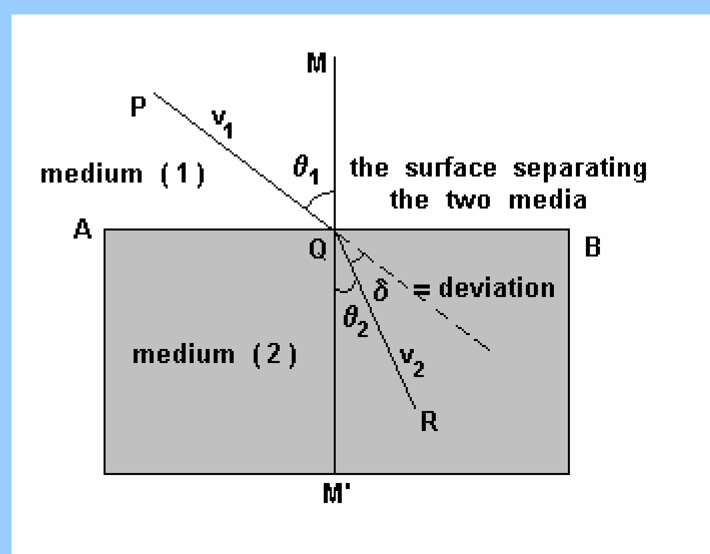
$$\therefore \quad n_{21} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

where v_1 and v_2 are the velocities of light in medium (1) and (2) respectively.

n_{21} depends upon (i) the type of the media, (ii) their temperatures and (iii) the wavelength of light.

The refractive index, n, of a medium with respect to vacuum (or in practice air) is called its absolute refractive index.

$\therefore \quad n = \frac{c}{v}$, where c is the velocity of light in vacuum and v its velocity in the medium.



In the figure, ray PQ is incident at an angle θ_1 to the normal drawn at point Q on the surface separating medium (1) and medium (2). QR is the refracted ray making an angle θ_2 with the normal.

Absolute refractive index of medium (1) and medium (2) are respectively,

$$n_1 = \frac{c}{v_1} \quad \text{and} \quad n_2 = \frac{c}{v_2},$$

$$\therefore \quad \frac{n_2}{n_1} = \frac{v_1}{v_2} = n_{21} = \frac{\sin\theta_1}{\sin\theta_2}$$

$$\Rightarrow \quad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

The ratio of the absolute refractive index of medium (2) to the absolute refractive index of medium (1) is the relative refractive index of medium (2) with respect to medium (1).

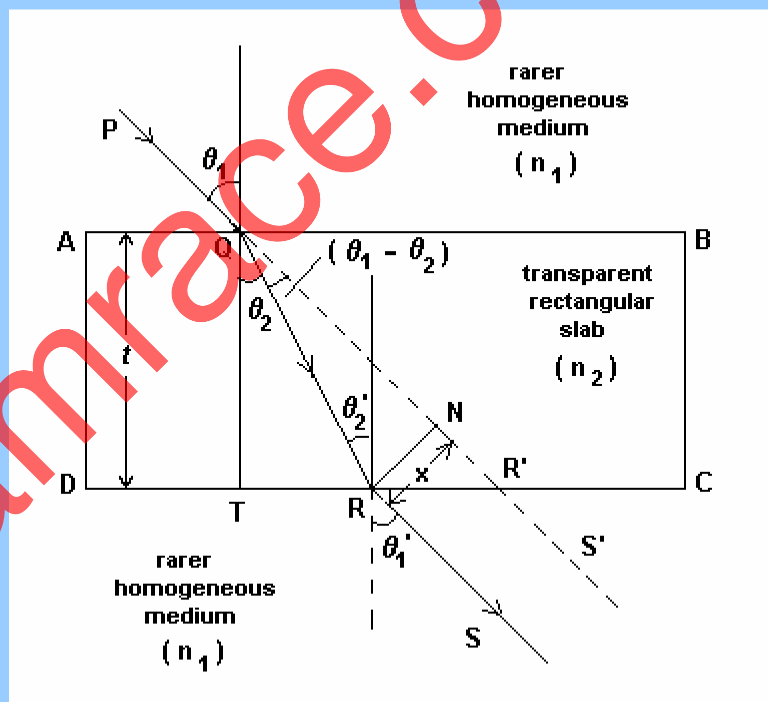
For the media shown in the figure, $n_2 > n_1 \Rightarrow \sin\theta_1 > \sin\theta_2 \Rightarrow \theta_1 > \theta_2$.

Thus, when a ray of light goes from a rarer medium to a denser medium, the angle of refraction is smaller than the angle of incidence and the ray bends towards the normal and when it goes from a denser medium to a rarer medium, it bends away from the normal.

As above, $n_{21} = \dfrac{v_1}{v_2}$. $\therefore$ $n_{12} = \dfrac{v_2}{v_1}$ $\Rightarrow$ $n_{21} \times n_{12} = 1$

This result can be generalized for any number of mediums.

Lateral shift



As shown in the figure, if a ray of light traveling in a rarer, homogeneous medium, remains in the same medium, it will move along the path PQR'S'. But if it enters into a rectangular slab of denser medium, it will get refracted twice at surfaces, AB and CD.

As the media on both sides of the rectangular slab is the same,

$n_{21} = 1 / n_{12}$ and $\theta_1 = \theta_1'$

Thus the emergent ray RS is parallel to the incident ray but due to refraction it shifts by an amount RN = x. Such a deviation of the incident ray is called lateral shift.

From the figure,

Lateral shift, $x = QR \sin(\theta_1 - \theta_2)$

$= QT \sec\theta_2 \sin(\theta_1 - \theta_2)$

$= t \sec\theta_2 \sin(\theta_1 - \theta_2)$

If θ_1 is very small, θ_2 is also small $\Rightarrow \sin(\theta_1 - \theta_2) \approx (\theta_1 - \theta_2)$ radian and $\sec\theta_2 \approx 1$

$$\therefore\ x = t(\theta_1 - \theta_2) = t\,\theta_1\left(1 - \frac{\theta_2}{\theta_1}\right)$$

Now according to Snell's law,

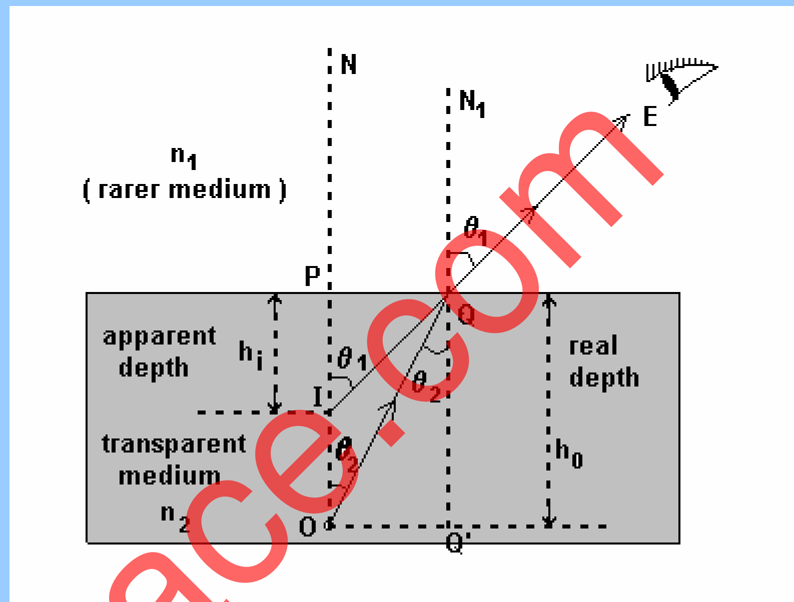
$$\frac{\theta_2}{\theta_1} \approx \frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2}$$

$$\therefore\ x = t\,\theta_1\left(1 - \frac{n_1}{n_2}\right)$$

Another example of refraction:

An object inside water when viewed from outside appears raised due to the phenomenon of refraction.

As shown in the figure, suppose an object is at real depth h_o in a denser medium (like water of refractive index, n_2). Ray OQ from O refracts at Q and reaches eye of the observer along QE. EQ extended in denser medium meets the normal PN at I. So the observer sees the image at I at an apparent depth, h_i.



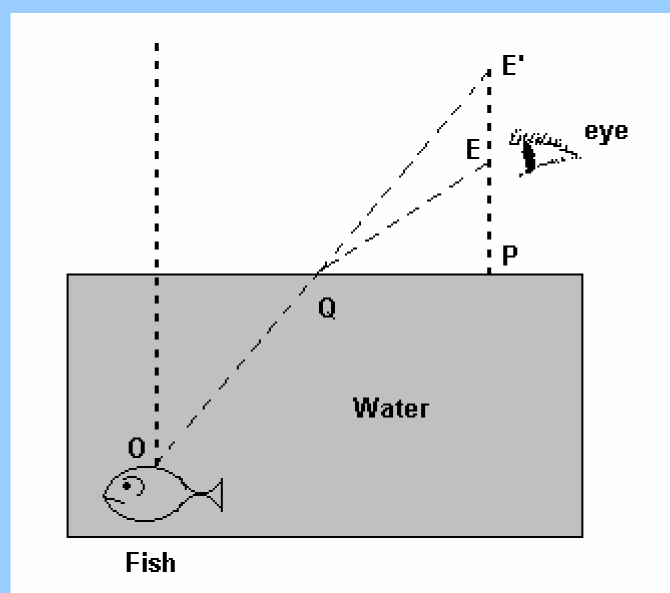
<u>If the angle of incidence, θ_1 is very small</u>, θ_2 will also be small.

$$\therefore \quad \sin\theta \approx \theta \approx \tan\theta$$

By Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow n_1 \tan\theta_1 = n_2 \tan\theta_2$

$$\therefore \quad n_1\left(\frac{PQ}{PI}\right) = n_2\left(\frac{PQ}{PO}\right) \Rightarrow \frac{\text{apparent height, } h_i}{\text{real height, } h_o} = \frac{n_1}{n_2} = \frac{n\,(\text{rarer})}{n\,(\text{denser})}$$

Now suppose that the observer is a fish inside the water and it views the eye of the person along OQ (as shown in the figure). Ray OQ extended in air meets the normal drawn from point E to the surface at E'. Thus the fish sees the person's eye at E' instead of E.



Here, EP = real height
and E'P = apparent height.

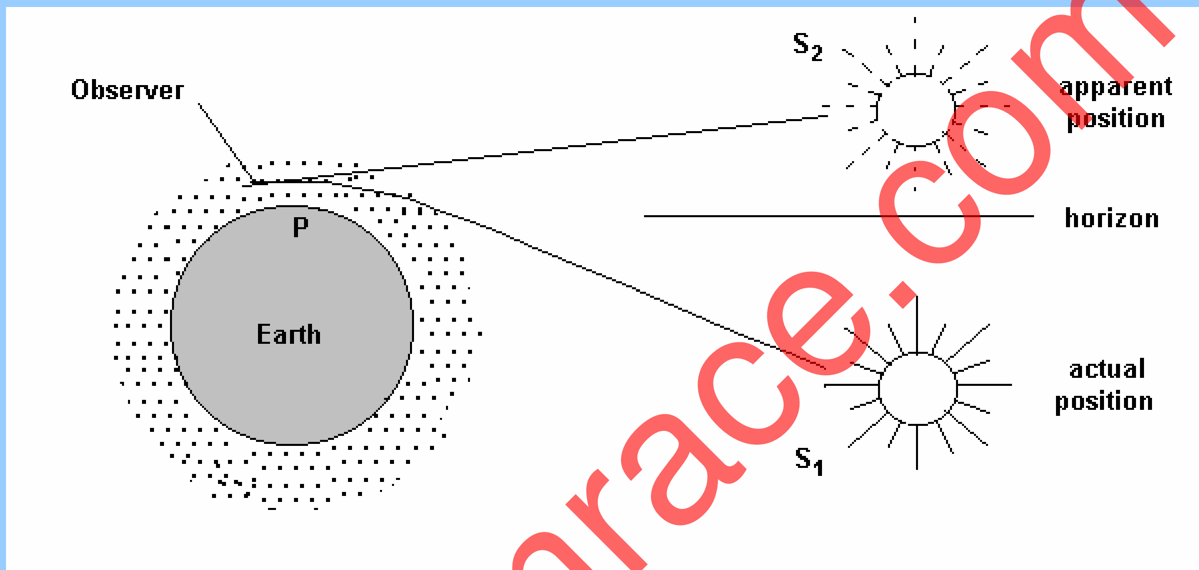
This example shows that if an object in a rarer medium is viewed from denser medium, it appears to be raised.

Thus, if an object is kept in a rarer medium, at height h_o from the interface and is viewed normally from the denser medium, then it appears to be at a height h_i ($h_i > h_o$) and in this case

$$\frac{\text{apparent height, } h_i}{\text{real height, } h_o} = \frac{n_2}{n_1} = \frac{n\,(\text{denser})}{n\,(\text{rearer})}$$

One more interesting case:

As we go higher in the Earth's atmosphere, it becomes optically rarer. Thus light coming from the sun and stars reaches the observer on Earth passing through the medium of continuously increasing refractive index and hence its direction continuously changes.



As shown in the figure, light rays from actual position, S_1, of the sun below the horizon reach the observer after continuous refraction in the Earth's atmosphere. The tangent to the curved path of the ray at point P passes through the apparent position, S_2, of the sun above the horizon.

Taking the refractive index of air as 1.00029, the apparent shift in the position of the sun is approximately half a degree which corresponds to a time interval of 2 minutes. Thus sunrise is seen 2 minutes earlier and sunset is seen 2 minutes later than the actual event.

10.5 Total Internal Reflection

Higher the refractive index of the medium, more is its optical density which is independent of the material density (mass / volume) of the medium.

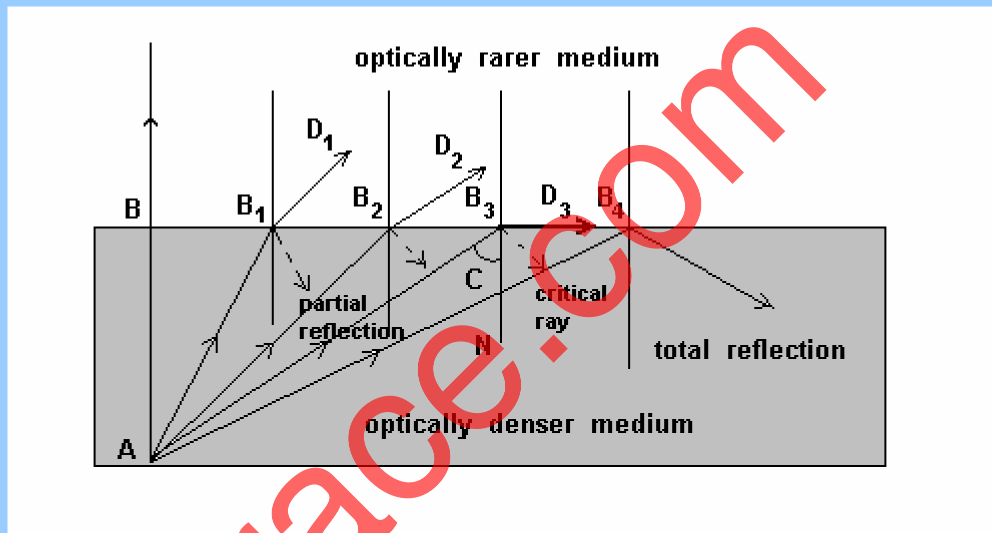
When light is refracted, it is partially reflected also. For a given intensity, I_0, of the incident light, the intensity of the reflected light, I_r, depends upon the angle of incidence. For normal incidence on a surface separating two media of refractive indices, n_1 and n_2, the intensity of reflected light is

$$I_r = I_0 \frac{(n_2 - n_1)^2}{(n_2 + n_1)^2}$$

For air (n = 1.0) and glass (n = 1.5), nearly 4 % of the incident energy is reflected.

Refer to the figure on the next page. A is a point object (or a light source) in a denser medium. Rays AB, AB_1, AB_2, ... undergo partial refraction and partial reflection at points of incidence B, B_1, B_2, etc. on the surface separating the two media. As the angle of incidence keeps on increasing, the angle of refraction also increases and for the incident ray AB_3, the refracted ray is along the surface separating two media, i.e., the angle of refraction is 90°.

"The angle of incidence for which the angle of refraction is 90° is called the critical angle, C, of the denser medium with respect to the rarer medium."



Using Snell's law,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

For critical angle of incidence,

$$\theta_1 = C \text{ and } \theta_2 = 90°$$

$$\therefore \; n_1 \sin C = n_2$$

$$\therefore \; \sin C = \frac{n_2}{n_1} = \frac{1}{n}$$

(taking the refractive index, n_2, of rarer medium air as 1 and of the denser medium, n_1, as n)

At the critical angle of incidence, the refracted ray is called the critical ray.

If the angle of incidence is more than the critical angle, there is no refraction and the incident ray gets completely reflected and its intensity also increases. This is known as total internal reflection and it obeys the laws of reflection.

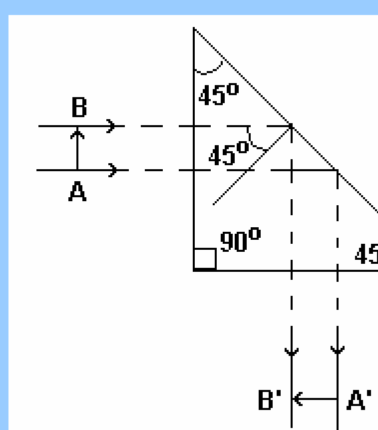
Uses of Total Internal Reflection:

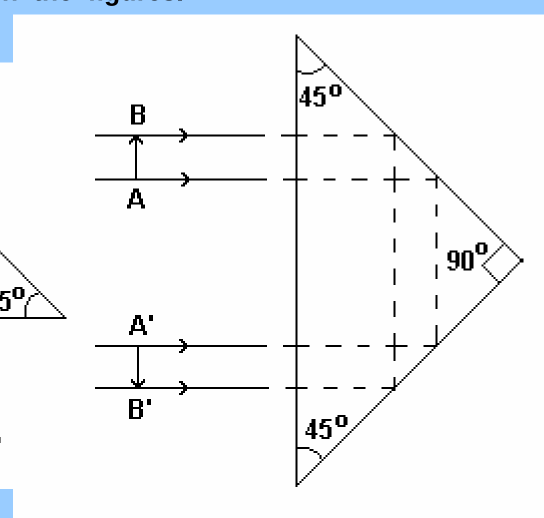
(1) The refractive index of diamond is 2.42 and its critical angle is 24.41°. Hence with proper cutting of its faces, light entering into it undergoes many reflections and the diamond sparkles.

(2) Using isosceles right angled prisms and taking advantage of total internal reflection, light can be deviated by 90° and 180° as shown in the figures.

As can be seen from the figures that in both cases, the critical angle of the prism w.r.t. to air must be less than 45°.

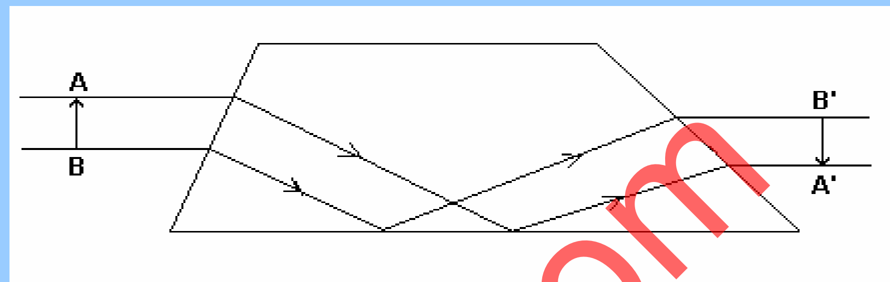
Prisms of crown glass (C = 41.14°) and flint glass (C = 37.31°) are used for this.





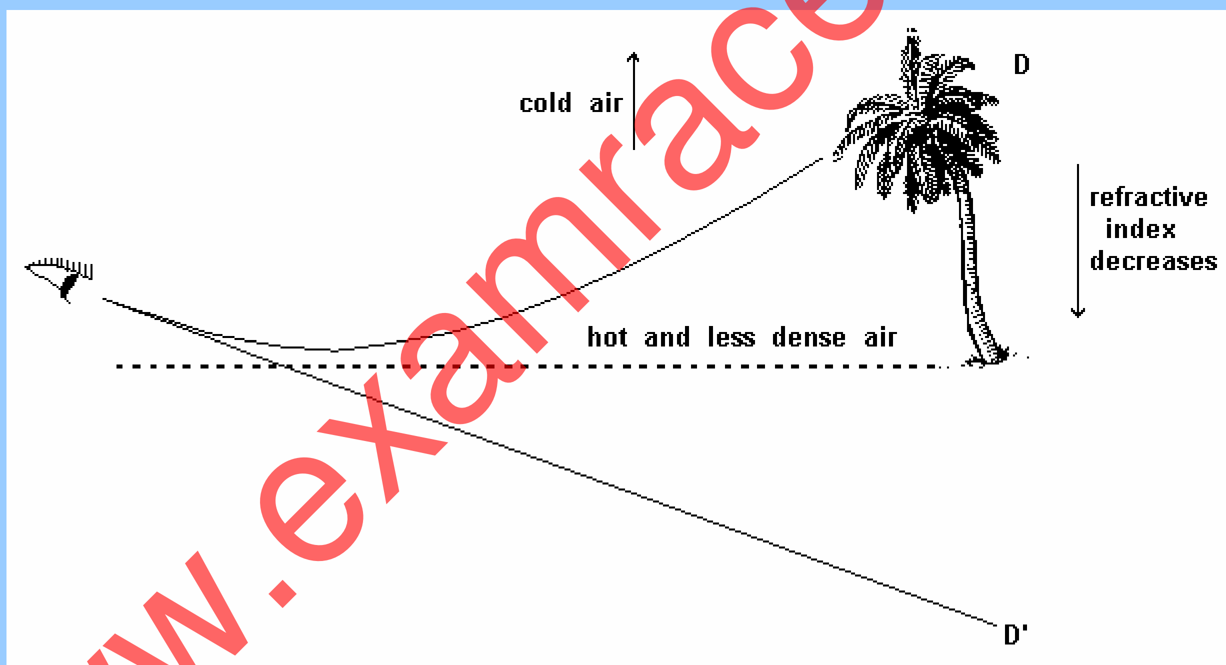
In the adjoining figure, the direction of light rays does not change but the image is inverted. This prism is called 'amici prism'.

In all the above cases, the size of the image remains the same as the size of the object.



(3) Mirage formation in hot regions during summer due to total internal reflection:

In summer, due to intense heat, the air in contact with the ground becomes hot and optically rarer as compared to the air above which is cold and optically denser.



As shown in the figure, a ray of light going from the top of the tree (D) to the ground travels continuously from a denser medium to a rarer medium. Its angle of incidence to the successive layers continuously increases due to refraction and when it exceeds the critical angle, the ray undergoes total internal reflection and reaches the eye of the observer. This ray appears to the observer as coming from a point D' directly below D as if it is coming from water. This kind of image formation is called a mirage (often seen in the deserts).

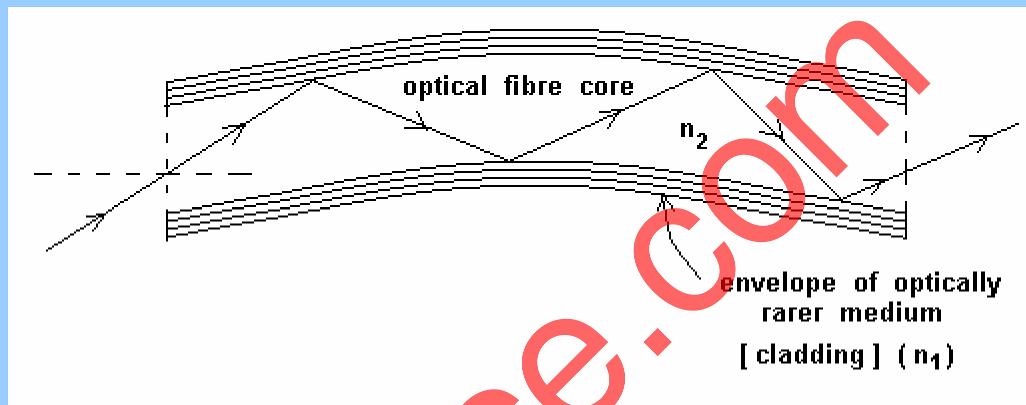
(4) Optical Fibres:

Optical fibres are long thin fibres made of glass or fused quartz of 10 to 100 μm diameter. The outer cladding of the fibres has a lower refractive index than the core of the fibre. The core (refractive index = n_2) and the cladding (refractive index = n_1) are so chosen that the critical angle of incidence is small.

As shown in the figure, a ray of light entering the optical fibre, entering at an angle of incidence greater than the critical angle, comes out of it after undergoing multiple total internal reflections.

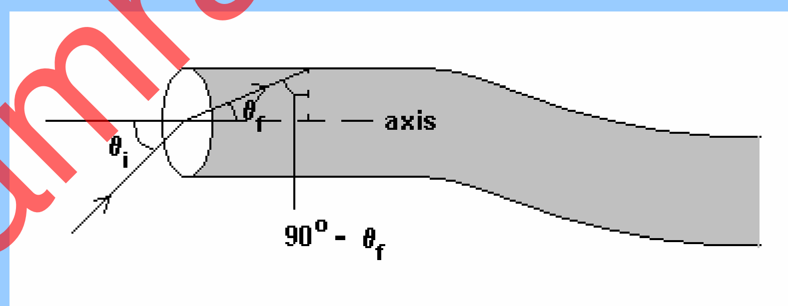
Even if the fibre is bent light remains within it. This is how endoscopy is done for viewing the human lungs, stomach, intestines, etc.

In communications, optical fibres are used to make distortionless signal transmission.



In absence of the cladding layer, due to dust particles, oil or other impurities, some leakage of light occurs. Fused quartz is used for making optical fibres because it is highly transparent.

As shown in the figure, a ray incident at an angle θ_i to the axis of the fibre is refracted at an angle θ_f.



This ray is incident on the wall of the fibre at an angle $90^\circ - \theta_f$ which if greater than the critical angle for fibre-air (or cladding) interface, will undergo total internal reflection.

Thus, $90^\circ - \theta_f > C \Rightarrow \sin(90^\circ - \theta_f) > \sin C \; (= 1/n)$,
where n = refractive index of the material of the fibre.

$\therefore \quad n \cos\theta_f > 1 \quad \dots \quad \dots \quad (1)$

According to Snell's law, $\sin\theta_i = n \sin\theta_f$

$$\therefore \quad n\cos\theta_f = \sqrt{n^2 - n^2\sin^2\theta_f} = \sqrt{n^2 - \sin^2\theta_i} > 1$$

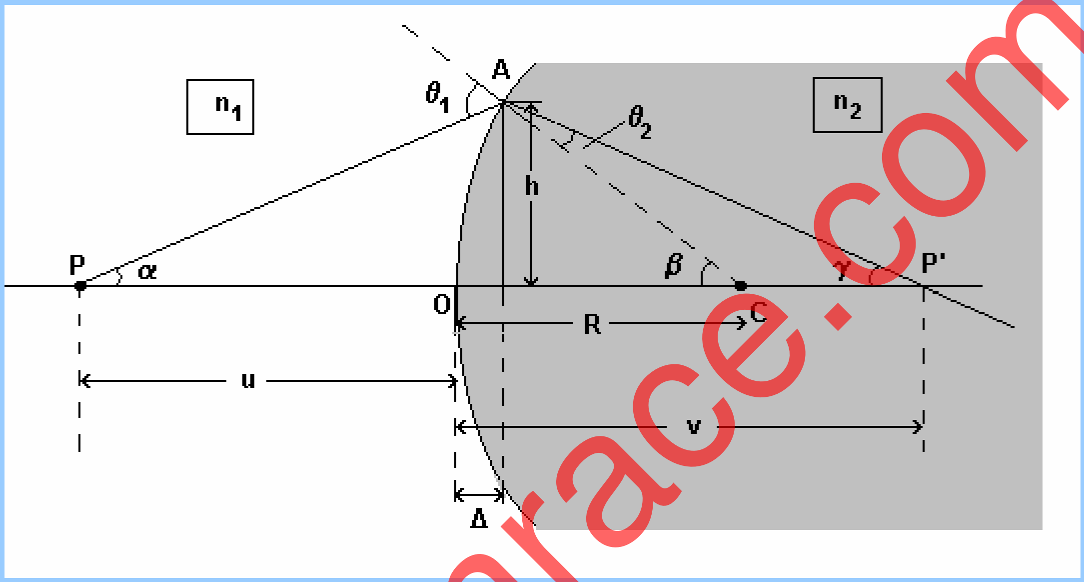
Now the maximum value of $\sin\theta_i = 1$. Hence if the above condition is satisfied for $\theta_I = 90^\circ$, it would be satisfied for any value of θ_i.

$$\therefore \quad \sqrt{n^2 - 1} > 1 \Rightarrow n > \sqrt{2}$$

Thus, if the value of refractive index is greater than $\sqrt{2}$, then the rays incident at any angle will undergo total internal reflection.

10.6 Refraction at a Spherically Curved Surface

As shown in the figure, a point object P is kept at a distance u from the centre, O, of the refracting curved surface on its axis OC. C is the centre of curvature of the refracting surface and R its radius. According to the sign convention, u is negative and R is positive.



(1) The ray PO is incident at O normal to the curved surface and hence travels undeviated along the axis OC of the curved surface.

(2) Another ray PA is incident at the point A on the curved surface at an angle θ_1 to the normal AC. Suppose the refractive index, n_1, of medium 1 is less than the refractive index, n_2, of medium 2. As a result, the refracted ray bends towards the normal and moves along AP'. θ_2 is the angle of refraction.

Both these rays meet at P' forming the image of P.

Applying Snell's law and noting that the angles θ_1 and θ_2 are small,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow n_1\theta_1 = n_2\theta_2 \quad \dots \dots \dots (1)$$

θ_1 is the exterior angle in Δ PAC, $\therefore \theta_1 = \alpha + \beta$
β is the exterior angle in Δ P'AC, $\therefore \theta_2 = \beta - \gamma$

Putting these values of θ_1 and θ_2 in equation (1) above,

$$n_1(\alpha + \beta) = n_2(\beta - \gamma) \quad \therefore \quad n_1\alpha + n_2\gamma = (n_2 - n_1)\beta \quad \dots \dots \dots (2)$$

As angles α, β and γ are small, using proper sign convention and neglecting Δ which is small as compared to u, v and R,

$$\alpha \approx \tan\alpha = \frac{h}{-u}, \quad \beta \approx \tan\beta = \frac{h}{R} \quad \text{and} \quad \gamma \approx \tan\gamma = \frac{h}{v}$$

Putting these values in equation (2),

$$n_1 \left(\frac{h}{-u} \right) + n_2 \left(\frac{h}{v} \right) = (n_2 - n_1) \frac{h}{R} \quad \Rightarrow \quad -\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

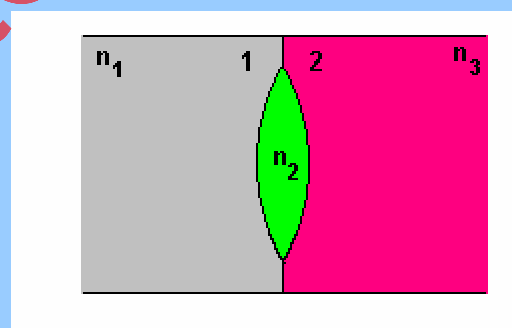
This equation is valid for a concave surface also. While using this equation proper sign convention has to be used.

If the image distance v is positive, then the refracted rays are to the right of origin O where they actually meet and the image has to be real. If the image distance v is negative, then the refracted rays are to the left of origin O where they can meet only by extending backwards and hence the image has to be virtual.

10.7 Thin Lenses

In general, a transparent medium bounded by two refracting surfaces is called a lens. The radii of curvature of the two refracting surfaces need not be equal. The lens for which the distance between the two refracting surfaces is negligible as compared to u, v and R is called a thin lens. For a thin lens, the distances can be measured from either surface.

Consider a convex lens as shown in the figure.



Radius of curvature of surface (1) = R_1
Radius of curvature of surface (2) = R_2

Refractive indices of :

medium to the left of the lens = n_1
material of the lens = n_2
medium to the right of the lens = n_3

(Contact lens is an example of different media on both sides of the lens. On one side is air and on the other side is the medium of the eye.)

For surface (1) $\quad -\frac{n_1}{u} + \frac{n_2}{v_1} = \frac{n_2 - n_1}{R_1} \quad \dots \dots \dots (1)$

For surface (1), R_1 is positive as it is to the right.

For surface (2), v_1 is the object distance and is positive as it is to the right of the surface.

For refraction by the second surface, the rays go from medium of refractive index n_2 to the medium of refractive index n_3.

$$\therefore \quad -\frac{n_2}{v_1} + \frac{n_3}{v} = \frac{n_3 - n_2}{R_2} \quad \dots \dots \dots (2)$$

Adding equations (1) and (2),

$$-\frac{n_1}{u} + \frac{n_3}{v} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

This is a general equation for a thin lens and is valid for concave lens also. If both sides of the lens has the same medium, then $n_1 = n_3$.

$$\therefore \quad -\frac{n_1}{u} + \frac{n_1}{v} = \frac{n_2 - n_1}{R_1} + \frac{n_1 - n_2}{R_2}$$

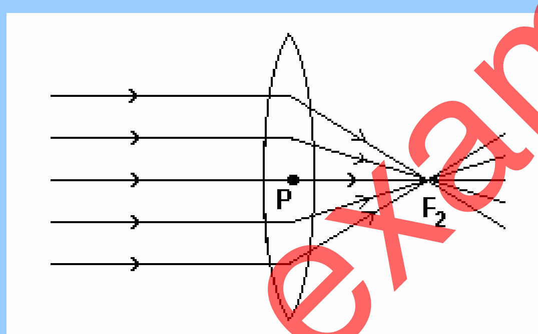
$$\therefore \quad n_1\left(\frac{1}{v} - \frac{1}{u}\right) = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

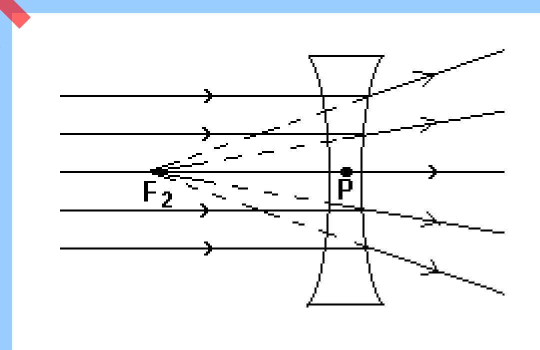
$$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Focal Point of a Thin Lens

A group of paraxial rays, incident on a convex lens, converge at a point F_2 after refraction on the other side of the lens. This point is called the principal focus of the lens and its distance from the optical centre, P, is the focal length f, of the lens.

For a concave lens, the incident paraxial rays are refracted away from the axis and when extended backwards meet at the principal focus, F_2 of the concave lens.





f is positive for a convex lens and negative for a concave lens. R_1 and R_2 have opposite signs for convex and concave lenses. Putting $u = \infty$ and $v = f$ in the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \quad \text{we get}$$

$$\frac{1}{f} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This is called lens-maker's formula as for a given material and given focal length, it gives the radii of curvature of the surfaces. Combining the above equations,

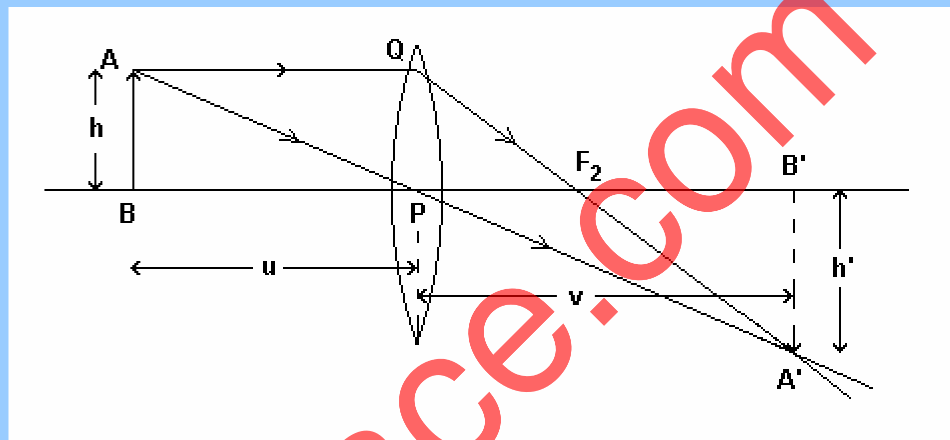
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This is called the Gaussian equation for a thin lens valid for both types of lenses.

Magnification of the image formed by a lens

As shown in the figure, AB is the object placed normal to the axis of the convex lens. Paraxial ray AQ after refraction passes through the principal focus F_2 on the other side. Ray AP passes through the optical centre of the lens and moves ahead as ray PA'.

Rays QF_2A' and PA' meet each other at point A' forming image of A. B' is the foot of normal from A' to the axis. Ray BP travels along the axis and after passing through the lens moves along path PB'. By symmetry, B' is the image of B and A'B' is the image of AB.



Now, magnification, $m = \frac{\text{height of image}}{\text{height of object}} = \frac{h'}{h} = \frac{v}{u}$

positive m $\Rightarrow$ erect and virtual image and negative m $\Rightarrow$ inverted and real image.

This formula can be used for a concave lens also.

Power of a Lens:

Converging capacity of a convex lens and diverging capacity of a concave lens is defined by the power of a lens. Power of a lens is the reciprocal of its focal length.

$\therefore$ **Power of a lens,** $P = \frac{1}{f}$

Power of a lens is so defined because a convex lens of a short focal length focuses the rays within a very short distance and hence its converging capacity or power is more. In the same way, lens of longer focal length has less power. Hence, the power of a lens is defined as the reciprocal of its focal length.

Power of a convex lens is positive as its focal length is positive and that of the concave lens is negative as its focal length is negative.

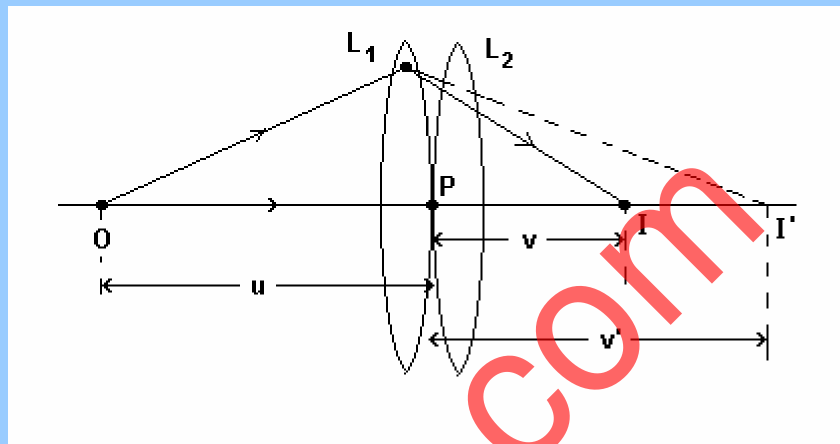
SI unit of power of a lens is dioptre. Its symbol is D. $1\,D = 1\,m^{-1}$

10.8 Combination of Thin Lenses in Contact

As shown in the figure (next page), two convex lenses L_1 and L_2 with focal lengths f_1 and f_2 are kept in contact in such a way that their principal axes coincide.

A point like object O is kept away from the principal focus of lens L_1 and its image due to this lens alone would have been at I'. This image behaves like a virtual object for lens L_2 and its image formed by lens L_2 is obtained at I.

As the lenses are thin, their contact point can be taken as the optical centre of the combination. Distances u, v and v' are shown in the figure.



For lens L_1, $\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1}$

For lens L_2, $\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2}$

Adding these two results, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$ ... ... (1)

If the focal length of the lens equivalent to the given combination of lenses is f, then

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ... ... (2)

From equations (1) and (2), $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ and, in general,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots\ldots\ldots\ldots + \frac{1}{f_n}$$

Here f is smaller than the smallest of f_1, f_2, f_3, ... f_n.

Power:

Replacing $\frac{1}{f} = P$, $\frac{1}{f_1} = P_1$, ... $\frac{1}{f_n} = P_n$ respectively in the above equation, we get

$P = P_1 + P_2 + P_3 + \ldots\ldots\ldots\ldots + P_n$

This sum is algebraic. P for some lenses (convex) will be positive and for some lenses (concave) will be negative.

From the figure above, magnification for lens L_1, $m_1 = \frac{v'}{u}$

magnification for lens L_2, $m_2 = \frac{v}{v'}$

and magnification for the lens-combination, $m = \frac{v}{u}$

$\therefore\ m = \frac{v}{u} = \frac{v}{v'} \times \frac{v'}{u} = m_2 m_1$

For a combination of more than two lenses,

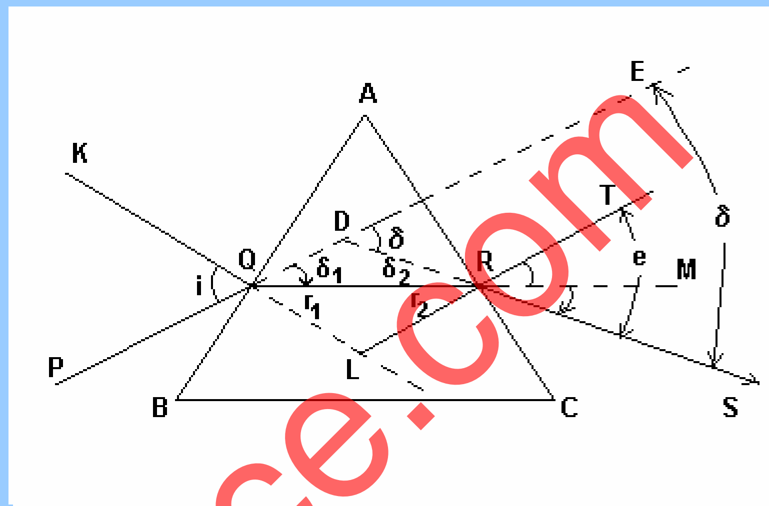
$m = m_1 \times m_2 \times m_3 \times \ldots\ldots\ldots\ldots \times m_n$

Position and Nature of Image formed by Mirrors / Lenses

Type of Mirror / Lens		Position of Object	Position of Image	Nature of Image
Concave Mirror OR Convex Lens	1	At infinity	At focus	Real inverted, Extremely diminished
	2	Beyond the centre of curvature (Mirror) Beyond 2f (Lens)	Between focus and centre of curvature (Mirror) Between f and 2f (Lens)	Real, Inverted Diminished
	3	At the centre of curvature (Mirror) At 2f (Lens)	At the centre of curvature (Mirror) At 2f (Lens)	Real, Inverted, Same Size
	4	Between focus and centre of curvature (Mirror) Between f and 2f (Lens)	Beyond the centre of curvature (Mirror) Beyond 2f (Lens)	Real, Inverted, Magnified
	5	At Focus	At infinity	Extremely Magnified
	6	Between the pole and principal focus (Mirror) Within f (Lens)	Behind the mirror, beyond the pole (Mirror) On the object side (Lens)	Virtual, Erect, Magnified
Convex Mirror OR Concave Lens	1	At infinity	At focus	Virtual, Erect, Diminished
	2	Between infinity and Mirror / Lens	Between the focus and the pole (Mirror) Between the lens and f	”

10.9 Refraction of Light Due to a Prism

The cross-section perpendicular to the rectangular surfaces of a prism, made up of a transparent material, is shown in the figure.



A ray of monochromatic light incident at point Q on the surface AB of the prism gets refracted and travels along QR undergoing deviation δ_1 at point Q. It is then incident at point R on the surface AC and emerges after refraction along RS undergoing deviation δ_2 at R.

When the emergent ray RS is extended backwards, it meets the extended incident ray PE in D. Angle, δ, between the incident and the emergent rays is called the angle of deviation.

As shown in the figure, in □ AQLR, $\angle AQL$ and $\angle ARL$ are right angles.

$$\therefore \quad \angle A + \angle QLR = 180^\circ \quad \ldots \ldots \ldots (1)$$

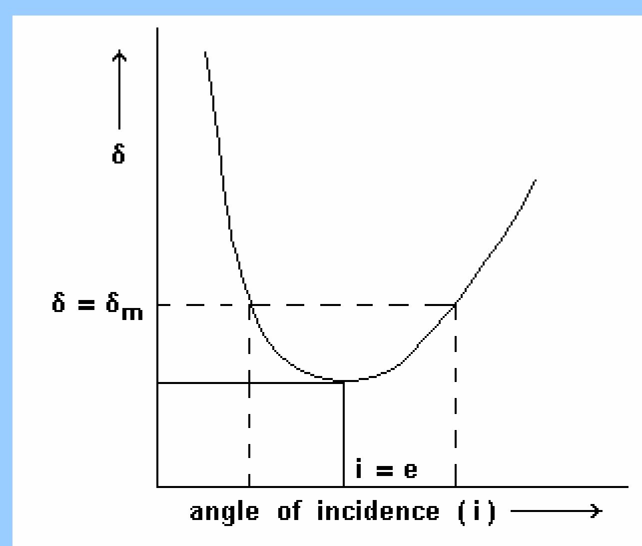
and in Δ QLR, $$r_1 + r_2 + \angle QLR = 180^\circ \quad \ldots \ldots \ldots (2)$$

$$\therefore \quad r_1 + r_2 + \angle A = 180^\circ \quad \text{(from equations (1) and (2))}$$

$$\therefore \quad r_1 + r_2 = A \quad \ldots \ldots \ldots (3)$$

In Δ DQR, exterior angle $$\delta = \delta_1 + \delta_2 = i - r_1 + e - r_2$$
$$= i + e - (r_1 + r_2)$$
$$\therefore \quad \delta = i + e - A \quad \ldots \ldots \ldots (4) \quad \text{(from equation (3))}$$

Thus, the angle of deviation depends on the angle of incidence. The graph of angle of deviation vs. angle of incidence for an equilateral prism is shown in the figure.



As can be seen from the graph, the angle of deviation is the same for two values of angle of incidence. This means that if the ray PQRS is reversed along path SRQP, i.e., if the angle of incidence is e, the angle of emergence will be i, but the angle of deviation will remain the same. As shown in the graph, for a particular value of the angle of deviation, $\delta = \delta_m$ which is minimum, there is only one angle of incidence.

Putting $\delta = \delta_m$ and $i = e$ in equation (4),

$$\delta_m = 2i - A$$

Using Snell's law for incident ray PQ at Q and SR at R,
$n_1 \sin i = n_2 \sin r_1$ and $n_1 \sin e = n_2 \sin r_2$

For minimum angle of deviation, $e = i$.

$\therefore \sin r_1 = \sin r_2 \Rightarrow r_1 = r_2 = r$ (suppose)

$$\therefore n_1 \sin i = n_2 \sin r_1 = n_2 \sin r = n_2 \sin\left(\frac{A}{2}\right) \quad \left(\because r = \frac{A}{2} \text{ from equation (3)}\right)$$

$$\therefore \frac{n_2}{n_1} = \frac{\sin i}{\sin r} = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]} \quad (\because \delta_m = 2i - A)$$

If the prism is kept in air, $n_1 = 1$ and $n_2 = n$,

$$\therefore n = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$

This equation shows that for a given prism, the value of δ_m depends upon (i) the angle of the prism, (ii) the refractive index of the material of prism and (iii) the refractive index of the medium in which the prism is kept.

When δ is minimum, the ray QR passing through the prism is parallel to the base BC of the prism (taking AB = AC).

Using the above equation we can calculate the refractive index of the prism with respect to the medium by measuring A and δ_m.

For a prism with small A, δ_m is also small.

Hence taking $\sin\left[\frac{A + \delta_m}{2}\right] \approx \frac{A + \delta_m}{2}$ (radian) and $\sin\left[\frac{A}{2}\right] \approx \frac{A}{2}$ (radian),

$$n = \left(\frac{A + \delta_m}{2}\right) / \left(\frac{A}{2}\right) = \frac{A + \delta_m}{A} \qquad \therefore \delta_m = A(n - 1)$$

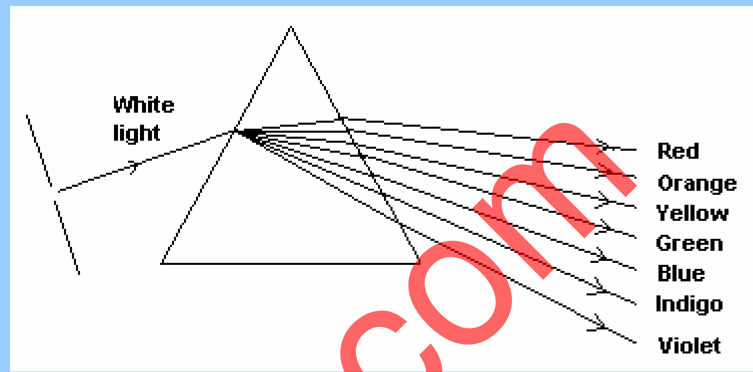
10.10 Dispersion of light due to a prism

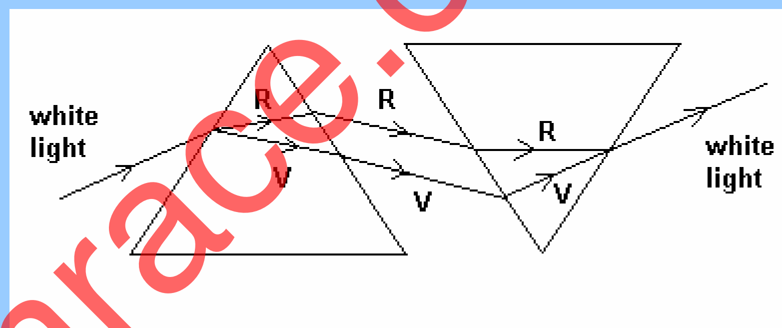
The phenomenon in which light gets divided into its constituent colours is known as dispersion of light.

As shown in the figure (next page), when a beam of white light or sun rays pass through a prism, the emergent light is seen to be dispersed into various colours.

Newton arranged two identical prisms of the same material as shown in the second figure and observed that when white light is incident on the first prism, emergent light from the second prism was also white. Hence it is clear that the first prism disperses the colours of white light and the second prism brings them together again to produce white light.

Visible light is made up of electromagnetic waves of wavelengths between 4000 Å and 8000 Å having different colours. All these waves have equal velocity in vacuum. Hence vacuum is called non-dispersive medium. But their velocities in some other medium of refractive index, n, are different. Such a medium is called a dispersive medium. So, as per n = c /v, the refractive indices of light having different wavelengths are different in a dispersive medium. For example, the velocity of violet light (v_V) is less than the velocity of red light (v_r). So the refractive index of violet light (n_V) is greater than the refractive index for red light (n_r). ($n_V > n_r$).





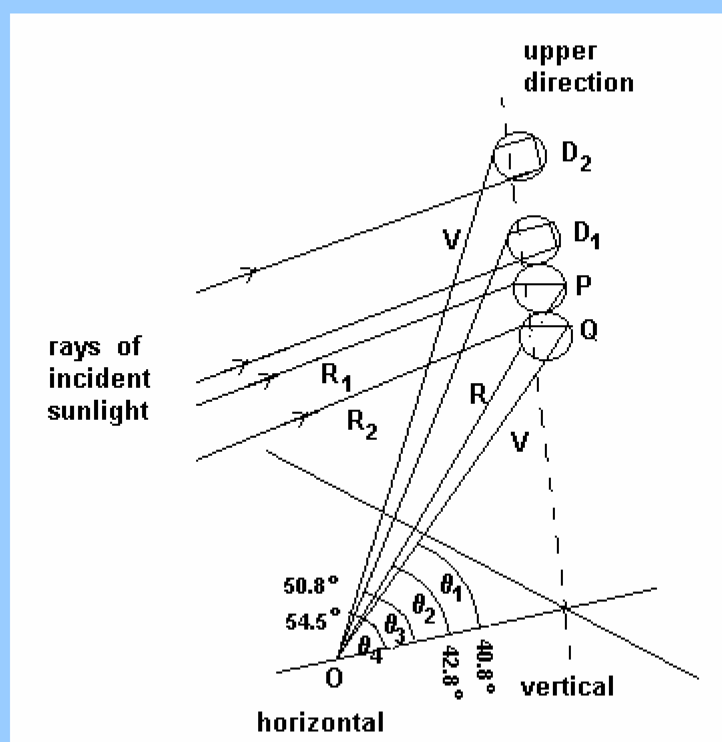
For other colours, values of refractive indices lie between n_r and n_V. If the angles of minimum deviation for red and violet colours are δ_R and δ_V respectively, then $\delta_R = A(n_r - 1)$ and $\delta_V = A(n_v - 1)$

$n_v > n_r \Rightarrow \delta_V > \delta_R$. Thus, violet colour deviates more than red colour.

For two given colours, the difference of their angles of deviation is known as the angular deviation corresponding to those colours. Value of $n_v - n_r$ is more for flint glass than for common crown glass. So, the spectrum obtained from such a prism is wider, more dispersed and more detailed.

10.11 Rainbow

Sunlight refracted and dispersed by the water droplets suspended in the atmosphere during monsoon form the rainbow pattern. Rainbow is a good example of dispersion and internal reflection of light.

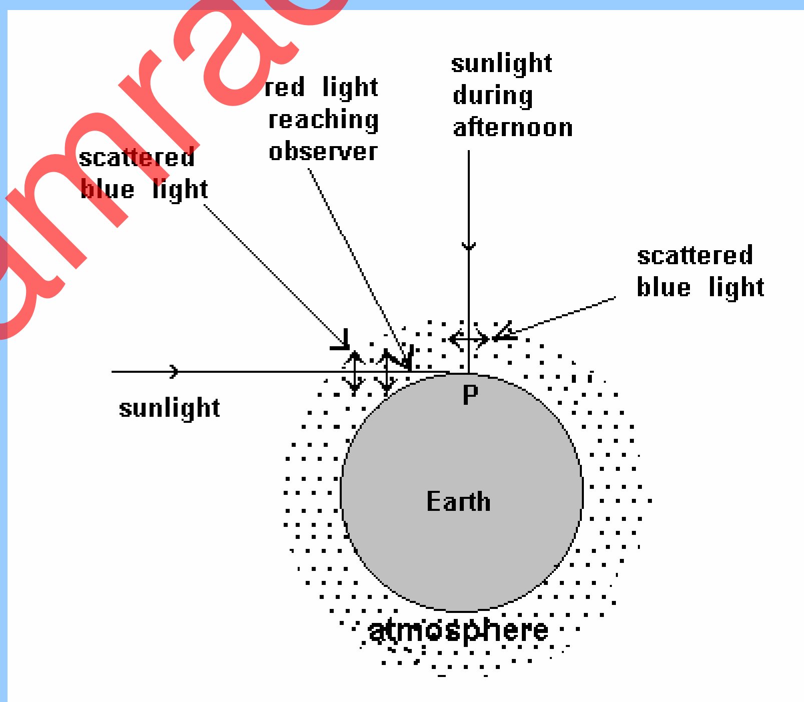


As shown in the figure, P and Q are two of the innumerable water droplets. Two rays R_1 and R_2 from the sun behind the observer incident on water droplets P and Q get refracted and dispersed. All colours undergo internal reflection and emerge after second refraction.

From the drop P, red colour light reaches the eye of the observer at an angle, θ_2 = 42.8°, with the horizontal. Thus, from all such droplets on the arc of a circle making this angle with the horizontal, red colour light reaches the eye of the observer. Similarly, from the drop Q, violet colour light reaches the eye of the observer at an angle θ_1 = 40.8°, with the horizontal and from all such droplets making this angle with the horizontal, violet colour light reaches the eye of the observer. All the remaining colours of light are seen between red and violet. Thus rainbow is seen in the form of a semicircle. This rainbow is known as primary rainbow. All the colours of a primary rainbow are accommodated within 2° near the eye.

Sometimes, a faint secondary rainbow is seen above the primary rainbow in which order of the colours gets reversed. Here, the internal reflection of light occurs twice as compared to once in the primary rainbow. The red and violet colours of secondary rainbow coming from water droplets D_1 and D_2 at an angle of θ_3 = 50.8° and θ_4 = 54.5° respectively are shown in the figure. All the colours of a secondary rainbow are accommodated within 3.7° near the eye. If the rainbow is observed from the height of a mountain or the top of a tower, some portion of rainbow below the horizon can also be seen.

10.12 Scattering of Light



Light incident on atmospheric atoms and molecules and small suspended particles like cloud droplets is absorbed by them and immediately reradiated in different directions in different proportions of intensity. This process is called the scattering of light.

If the size of the particle which scatters the light is smaller than wavelength of the incident light, the scattering is known as Rayleigh's scattering. Rayleigh observed that the scattering of light is inversely proportional to the fourth power of the wavelength of light. As the wavelength of blue light is 1.7 times smaller than that of red light, it scatters 8 to 9 times more than the red light. Although violet and indigo light also have short wavelengths, their proportion in light is much less and our eyes are not sensitive to these colours. So their scattering is not so important. The light having wavelength close to the wavelength of yellow colour has maximum intensity and even our eyes are more sensitive to the light of these colours.

The figure shows the light of the rising sun reaching the earth's atmosphere. In this condition, white light has to travel more distance through the atmosphere during which light of most of the colours is scattered and only red colour reaches the observer on the earth. So the sun appears reddish and the sky above appears bluish due to scattered blue colour. Similar situation prevails at the time of sunset also. The same reason is responsible for a reddish full-moon, while rising or setting.

If the size of particles due to which light is scattered are larger than the wavelength, the

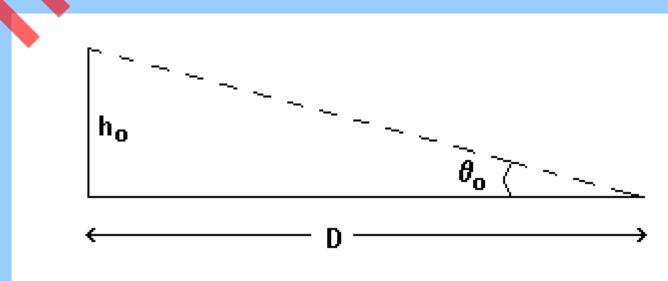
scattering is known as Mie-scattering. This type of scattering was first studied by Gustav Mie in 1908 A.D. In this type of scattering, the relation between the intensity and wavelength of the scattered light is complicated. However, as the size of the particle increases, the proportion of diffused reflection also increases. Size of the water particles forming white clouds being large, diffused reflection of sun-light takes place. As the diffused reflection is independent of the wavelength, all the wavelengths of visible light are reflected and so the clouds appear white.

10.13 Optical Instruments

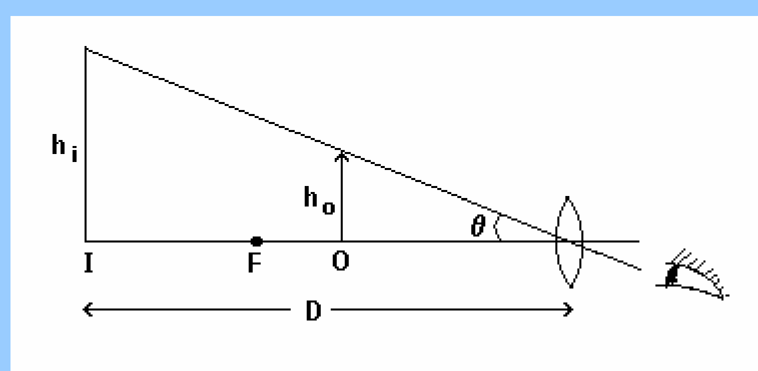
10.13 (a) Simple Microscope:

Apparent size of an object as seen by us depends on the actual size of the object and the angle subtended by it with our eye. When we see the railway track standing between the tracks, at far distance rails seem to be meeting each other. This is because the rays coming from the points far away on the rails, one from each rail subtend a very small angle with our eye. For this reason, to see a microscopic object clearly, we tend to keep it very near our eyes. But this strains the eyes and we can't see the object clearly. In fact, to see any object clearly without straining the eyes we have to keep it at some minimum distance from the eyes. This minimum distance is called the near point or the distance of most distinct vision. Hence to see a microscopic object clearly we place it within the focal length of a convex lens so that its virtual, magnified, erect image is formed at a comfortable distance of distinct vision and we can see it clearly. This convex lens is known as simple microscope.

Suppose a linear object with height h_o, kept at the near point (≈ 25 cm) from our eye, subtends an angle θ_o with our eye as shown in the figure.



Now, suppose the object is kept at such a distance within the focal length, f, of a convex lens that its virtual, erect and magnified image is formed at the near point as shown in the figure.



Here, as the eye is very near the lens, the angle θ, subtended by the object and image with the lens is the same as the angle subtended with the eye.

According to the definition:

Angular magnification or Magnifying power of the lens (M)

$$= \frac{\text{Angle subtended by the image, } \theta, \text{ on the near point with the eye}}{\text{Angle subtended by the object, } \theta_0, \text{ on the near point with the eye}} = \frac{h_i / D}{h_o / D}$$

(∵ θ and θ_o are very small)

$$= \frac{h_i}{h_o}$$

Magnifying power is same as linear magnification.

$$\therefore \quad m = \frac{v}{u} = \frac{D}{u} \qquad (\because\ v = D, \text{ the distance of most distinct vision})$$

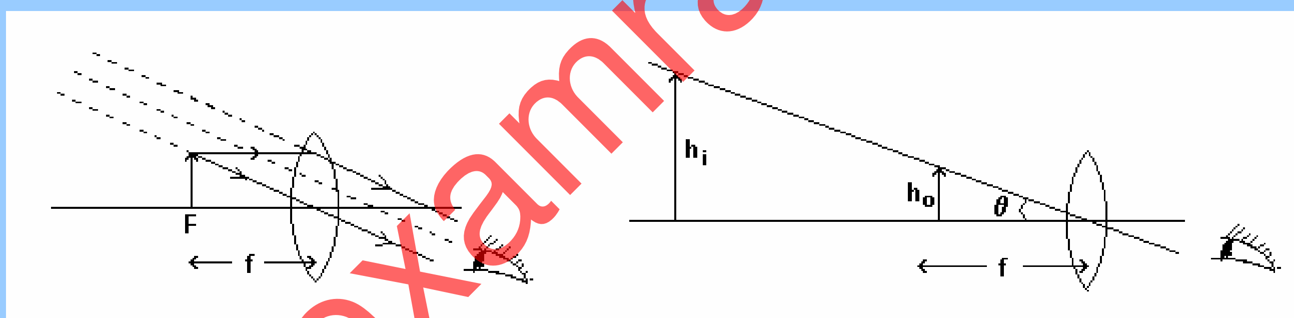
$$\text{Now,} \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \therefore \quad -\frac{1}{D} + \frac{1}{u} = \frac{1}{f} \qquad (\because\ v = D \text{ and } u \text{ are negative})$$

$$\therefore \quad m = \frac{v}{u} = 1 + \frac{D}{f}$$

If the image is at a very large distance (theoretically at infinity), magnification will be very large and 1 can be neglected as compared to D / f.

Thus the value of m would be between $\frac{D}{f}$ and $1 + \frac{D}{f}$.

To obtain the enlarged, clear object which can be seen without straining the eyes, the object should be kept near f, but at a distance less than f.



when object is within focal length f

when object distance is close to f but less than f (u < f)

10.13 (b) Compound Microscope:

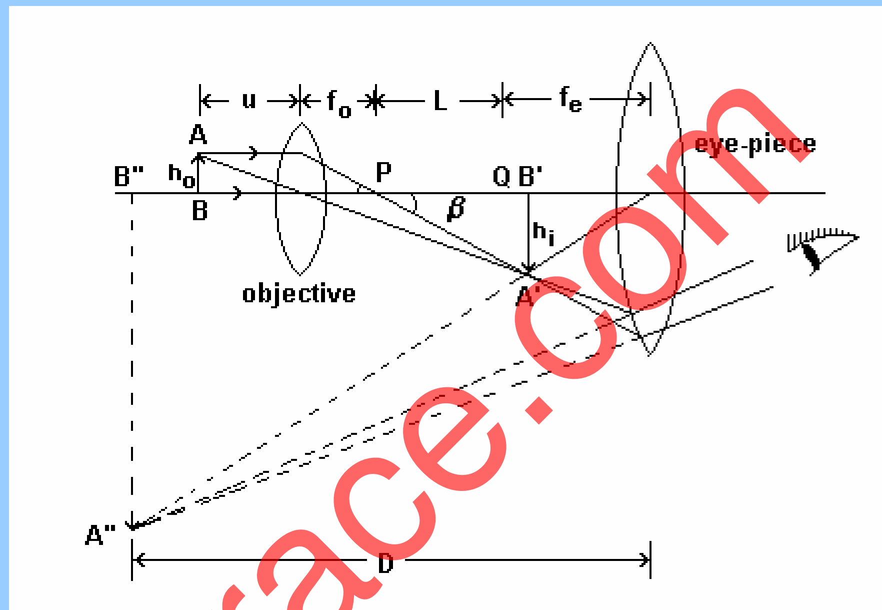
In a simple microscope, magnifying power depends on D / f. So to obtain more magnification, we may be tempted to use a small focal length. But this distorts the image. Hence, f cannot be taken very small and a simple microscope gives a maximum magnification of 20 X. Now if we use this magnified image as an object for another convex lens, we can further magnify it. Thus a compound microscope is made using two convex lenses.

Ray diagram for a compound microscope is shown in the figure on the next page.

The lens kept near the object is called 'objective' and the lens kept near the eye is known as 'eye-piece'. Distance between the second focal point (P) of the objective and the first focal point (Q) of the eye-piece is known as 'tube-length (L)' of the microscope.

As can be seen from the figure, the real, inverted and magnified image obtained by the objective near the focal-point (Q) of the eye-piece acts as an object for the eye-piece which behaving as a simple microscope gives a virtual and highly magnified final image at a very large distance.

Magnification obtained by a compound microscope



From the figure,

magnification due to the objective,

$$m_o = \frac{h_i}{h_o} = \frac{L}{f_o}$$

($\because$ $h_i \approx PQ \tan\beta = L \tan\beta$; $h_o = f_o \tan\beta$), where

h_i = size of the first image,
f_o = focal length of objective

Now, magnification due to the eye-piece,

$$m_e = \frac{D}{f_e},$$ where

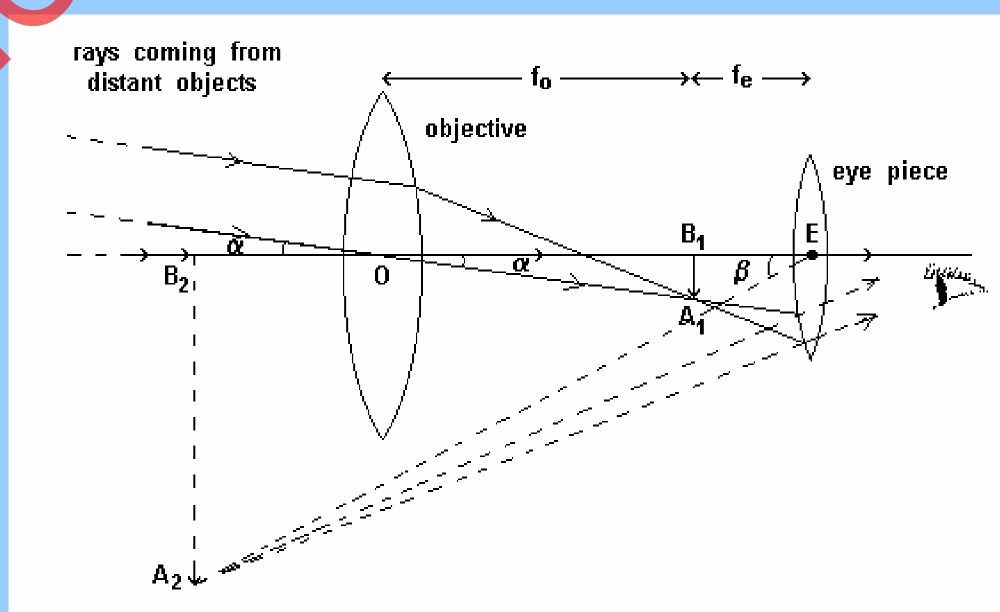
f_e = focal length of the eye-piece.

$\therefore$ magnification of the compound microscope, $m = m_o m_e$

$$\therefore \; m = \frac{L}{f_o} \times \frac{D}{f_e}$$

10.13 (c) Astronomical Telescope:

Astronomical Telescope is used to observe very huge celestial bodies and stars which are far away from us and from each other.



Its ray diagram is shown in the figure. Here two convex lenses are kept on the same principal axis. The lens facing the object is called objective whose diameter and focal length are greater than the lens known as eye-piece kept near the eye.

When the telescope is focused on a distant object, parallel rays coming from this object form a real, inverted and small image A_1B_1 on the second principal focus of the objective. This image is the object for the eye-piece. Eye-piece is moved to or fro to get the final and magnified inverted image A_2B_2 of the original object at a certain distance.

Magnification of the telescope,

$$m = \frac{\text{Angle subtended by the final image with eye}}{\text{Angle subtended by the object with the objective or eye}} = \frac{\beta}{\alpha} = \frac{A_1B_1}{f_e} \times \frac{f_o}{A_1B_1}$$

$$\therefore \quad m = \frac{f_o}{f_e}$$

Hence, to increase the magnification of the telescope, f_o should be increased and f_e should be decreased. $f_o + f_e$ is the optical length of the telescope. So, length of the telescope, $L \geq f_o + f_e$.

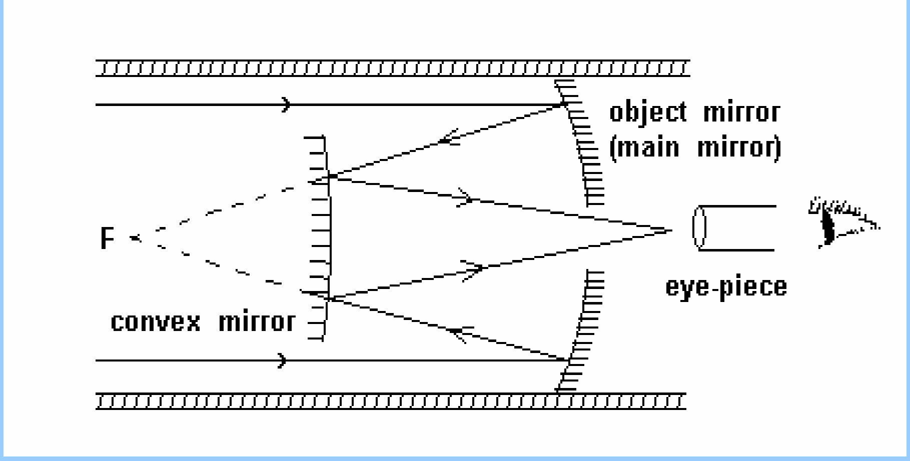
If f_o = 200 cm and f_e = 1 cm; m = 200. Using such a telescope, if the stars having angular distance 1' are observed, they would be seen at 200' = 3.33° angular distance between them.

For a telescope light gathering power and resolving power (power to view two nearby objects distinctly) are very important. Amount of light entering the objective of the telescope is directly proportional to the square of the diameter of the objective. With increase in the diameter of the objective, resolving power also increases.

In the telescope described above, rays from the object are refracted by the objective to form the image. Such a telescope is called a refractive telescope. Image formed in this type of telescope is inverted. To get rid of this problem, there is an extra pair of inverting lenses in the terrestrial telescope so that the erect image of the distant object is obtained.

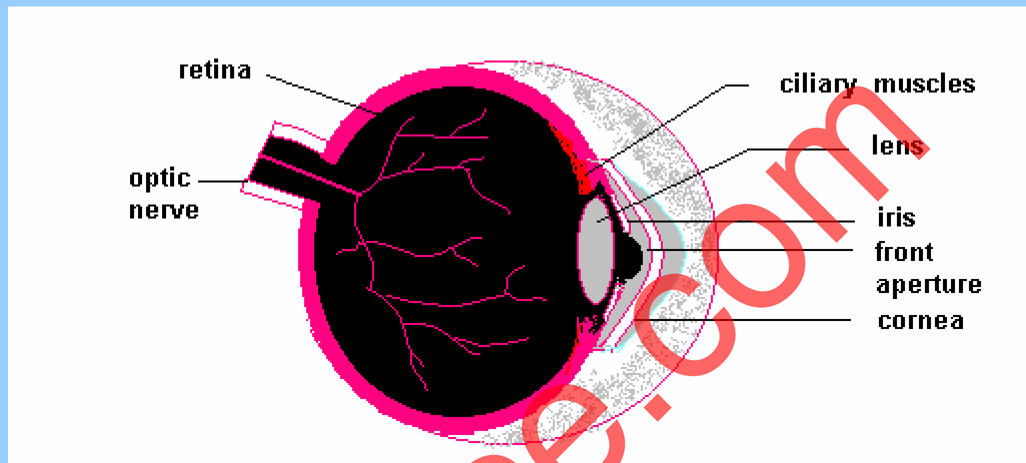
For better resolution and magnification, mirrors are used in modern telescopes. Such a telescope is known as a reflecting telescope. In such a telescope, problems of chromatic and spherical aberration are also overcome if a parabolic mirror is used. Construction of such a telescope is shown in the following figure.

Parallel rays coming from a distant object are incident on the reflecting surface of the primary parabolic concave mirror. A convex mirror is kept in the path of the reflected rays which would have focused at F forming the image. Rays reflected by the secondary mirror are focused on the eye-piece after passing through the hole kept in the primary mirror. Diameter and focal length of the primary mirror are kept large in such a telescope.



10.13 (d) Human Eye:

As shown in the figure, the ray entering the eye is first refracted in the cornea and then in the eye lens which is the main refractor. This forms inverted, real image on the retina which is processed in the brain and a final erect image is seen.



Retina has two types of cells:

(1) Rods: These cells receive the sensations of less intense light.

(2) Cones: These cells receive the sensation of colour and more intense light.

In the eye, the distance between the retina and the lens is fixed. Hence to form the image of objects at different distances exactly on the retina, focal length of the lens has to be changed. This is done by ciliary muscles which make the lens thick or thin as required. The iris controls the amount of light entering the eye by controlling the size of the pupil (front aperture). When we see the object kept on the side, lens of the eye rotates and brings the image on the central region of the retina (fovea).

Defects of Vision

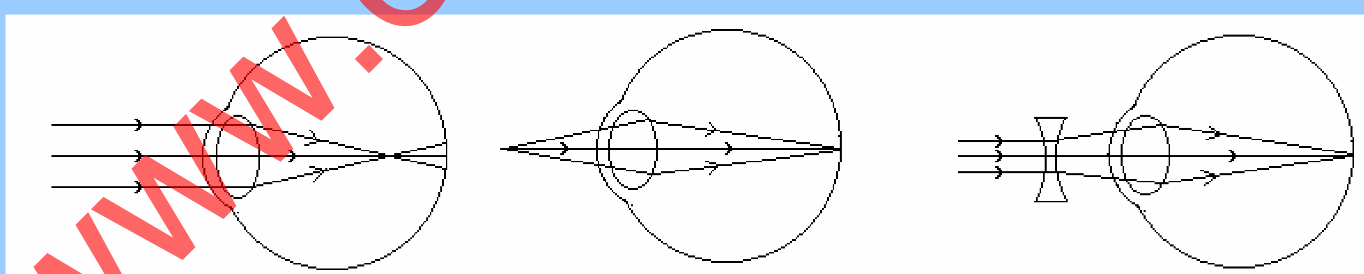
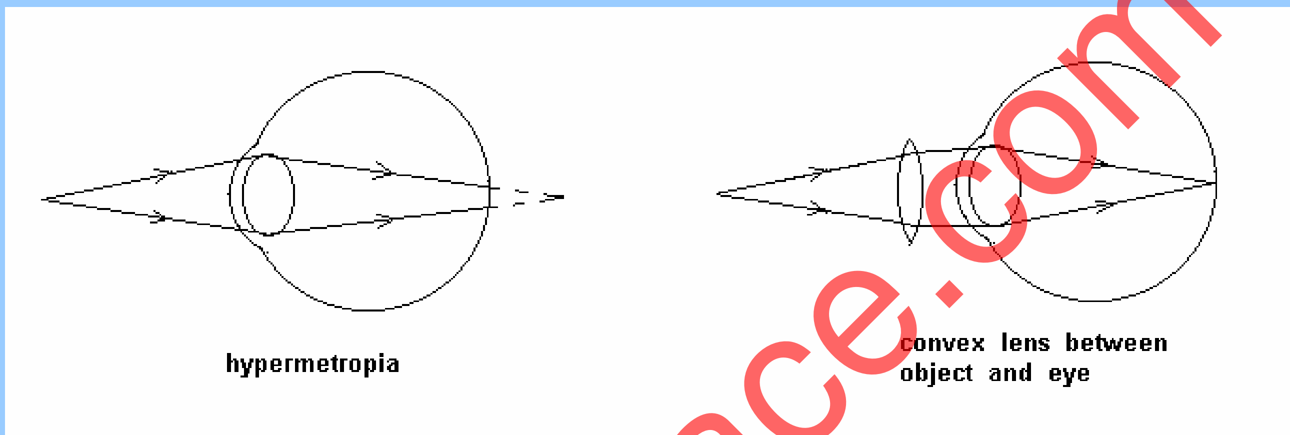


Image of distant object formed in front of retina | Image of nearby object formed on the retina | concave lens between object and eye

If the thickness of the eye lens cannot be altered as needed, then the rays coming from distant objects which are parallel, undergo extra refraction and focus in front of the retina as shown in the first figure. So distant objects cannot be seen clearly. But the image of the nearby object is formed on the retina as shown in the second figure. This type of defect is called **'near sightedness (myopia)'**.

To correct this defect, concave lenses are used as shown in the third figure above.

If the lens remains thin and does not become thick as needed, the rays from a nearby object undergo less refraction and focus behind the retina as shown in the first figure. Such an image cannot be seen clearly. Image of a distant object is formed on the retina and can be seen clearly, but nearby objects cannot be seen clearly. This type of defect is called **'far sightedness (hypermetropia)'.**

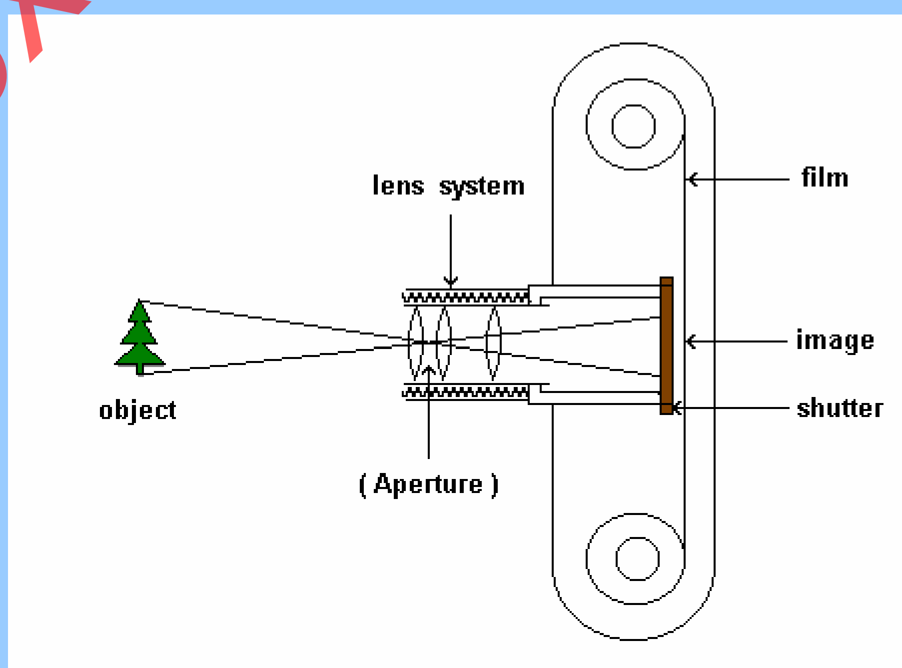


This type of defect is due to less convergence of rays and can be corrected by using a convex lens of proper focal length as shown in the second figure above.

Some persons can see only horizontal or vertical wires in a wire mesh clearly but not both. This defect is called **'astigmatism'.** This defect is due to unequal curvatures of the lens and cornea. Here horizontal curvatures are same but not the vertical. So rays are refracted equally in the horizontal plane but unequally in the vertical plane. As a result horizontal wires are seen clearly, but not the vertical wires. To remove this defect, cylindrical lenses are used.

10.13 (e) Photographic Camera:

As shown in the figure, in a photographic camera a combination of 3 convex lenses at one end and a photo sensitive surface at the other end are kept in a light proof box.



When a photograph is taken the shutter opens and shuts quickly. Light enters through the lens and is incident on the film during the time when the shutter remains open. Thus, due to the lens, a real and inverted image of the object is formed on the film. The amount of light entering the camera is controlled using the aperture of the lens.

The distance between the lens and the film can be adjusted for better pictures.

As the focal length of a lens in a camera is small (app. 50 mm), the changes required in the distance between the lens and the film are very small even for large object distance.

For clear and good quality photographs, the following points are important.

(1) Exposure Time:

The time for which the light is incident on the film is known as the exposure time. Less exposure time is kept in sunlight or more light. For indoor photography amount of light is less, so more exposure time is kept. For fast moving objects, less exposure time is kept. For a given aperture in a camera, usual exposure times are $\frac{1}{500}$ s, $\frac{1}{250}$ s, $\frac{1}{125}$ s, $\frac{1}{60}$ s, $\frac{1}{30}$ s,...

(2) Aperture of the Lens:

Diameter of the circular passage of light kept in a camera is the aperture. Some useful apertures known as f-number are $\frac{f}{2}$, $\frac{f}{2.8}$, $\frac{f}{4}$, $\frac{f}{5.6}$, $\frac{f}{8}$, $\frac{f}{11}$, $\frac{f}{16}$,... where f is the focal length of the lens. If the film is exposed equally for aperture diameter, $d_1 = \frac{f}{4}$ and $d_2 = \frac{f}{8}$, then for diameter d_1, the shutter should be kept open for 4 times longer compared to that for diameter d_2. Thus exposure time is inversely proportional to the area of the aperture.

(3) Speed of the film:

How quickly the film can be exposed is known as its speed. Fast film needs less exposure time and is used when light is less whereas slow film needs more exposure time and is suitable for the still photography.

(4) Exposure meter:

Some cameras are furnished with exposure meters which consists of a photosensitive surface. Electric current is produced in accordance with the intensity of light incident on the photosensitive surface which automatically adjusts the aperture and exposure time.

(5) Depth of Focus or Depth of Field:

If an object at a distance u can be perfectly focused on a film, Δu is the depth of focus which means that all objects within Δu distance of the object can be satisfactorily focused. More the aperture size, less is the depth of focus.

10.13 (f) Spectrometer:

Spectrometer is used in the laboratory to get a clear spectrum and determine the refractive index of the material of the prism.

It consists of a collimator, telescope and prism table. There is the scales below the prism table and the base of the spectrometer and leveling screws at the base. In the collimator, light entering through the adjustable slit is made parallel and incident on the refracting surface of the prism kept in a specific position on the prism table. Telescope is arranged to receive the refracted light and its eye-piece is moved to focus and get a clear spectrum of

the original light. We can know the angular position (θ) of the spectral lines of various colours by using a cross-wire and can be read from the scale. Prism table and telescope are arranged such that the angle of minimum deviation (δ_m) is obtained for each colour. Thus refractive index for the material of prism for a particular wavelength can be determined using the formula

$$n = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$

To measure the angle of the prism, A, prism table and telescope are so arranged that the rays refracted from one surface forming angle A enters the telescope and the same happens for the second surface. By knowing the angular position of the telescope in these two cases, angle of the prism can be calculated. The complete path of light is shown in the figure below.