

- (1) Find the parametric equations of the line passing through A (3, -2) and B (-4, 5) and hence express  $\overleftrightarrow{AB}$ ,  $\overrightarrow{AB}$  and  $\overline{AB}$  as sets.

$$\left[ \begin{array}{l} \text{Ans: Parametric equations of } \overleftrightarrow{AB} \text{ are } x = -7t + 3, y = 7t - 2, t \in \mathbb{R}. \\ \text{Further } \overleftrightarrow{AB} = \{(-7t + 3, 7t - 2) \mid t \in \mathbb{R}\}, \\ \overrightarrow{AB} = \{(-7t + 3, 7t - 2) \mid t \geq 0, t \in \mathbb{R}\} \\ \text{and } \overline{AB} = \{(-7t + 3, 7t - 2) \mid 0 \leq t \leq 1, t \in \mathbb{R}\} \end{array} \right]$$

- (2) If the length of the perpendicular segment from the origin is 10 and  $\alpha = -\frac{5\pi}{6}$ , then find the equation of the line.

$$[\text{Ans: } \sqrt{3}x + y + 20 = 0]$$

- (3) If the lines  $3x + y + 4 = 0$ ,  $3x + 4y - 5 = 0$  and  $24x - 7y - 3 = 0$  contain the sides of a triangle, prove that the triangle is isosceles.

- (4) Find the co-ordinates of the point at a distance of 10 units from the point (4, -3) on the line perpendicular to  $3x + 4y = 0$ .

$$[\text{Ans: } (10, 5), (-2, -11)]$$

- (5) A  $(x_1, y_1)$  and B  $(x_2, y_2)$  are points of the plane. If the line  $ax + by + c = 0$  divides  $\overline{AB}$ , find the ratio in which it divides  $\overline{AB}$  from A.

$$\left[ \text{Ans: } \lambda : 1 = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}, \quad ax_2 + by_2 + c \neq 0 \right]$$

- (6) If the sum of the intercepts on the axes of a line is constant, find the equation satisfied by the mid-point of the segment of the line intercepted between the axes.

$$[\text{Ans: } x + y = k, \text{ where } 2k = \text{constant sum of the intercepts}]$$

- (7) Find k if the lines  $kx - y - 2 = 0$ ,  $2x + ky - 5 = 0$  and  $4x - y - 3 = 0$  are concurrent.

$$[\text{Ans: } k = 3 \text{ or } -2]$$

- (8) Among all the lines passing through the point of intersection of the lines  $x + y - 7 = 0$  and  $4x - 3y = 0$ , find the one for which the length of the perpendicular segment on it from the origin is maximum.

[ Ans:  $3x + 4y - 25 = 0$  ]

- (9) Prove that the product of the perpendicular distances of the line  $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$  from the points  $\left( \pm \sqrt{a^2 - b^2}, 0 \right)$  is  $b^2$ .

- (10) Prove that if  $l m_1 \neq l_1 m$ ,  $n \neq n_1$  and  $l^2 + m^2 = l_1^2 + m_1^2$ , then the lines  $lx + my + n = 0$ ,  $l_1x + m_1y + n_1 = 0$ ,  $lx + my + n_1 = 0$  and  $l_1x + m_1y + n = 0$  form a rhombus.

- (11) Prove that the lines  $(a^2 - 3b^2)x^2 + 8abxy + (b^2 - 3a^2)y^2 = 0$  and  $ax + by + c = 0$ ,  $c \neq 0$  contain the sides of an equilateral triangle whose area is  $\frac{c^2}{\sqrt{3}(a^2 + b^2)}$ .

- (12) Two lines are represented by  $3x^2 - 7xy + 2y^2 - 14x + 13y + 15 = 0$ . Find the measure of the angle between them and the point of their intersection.

[ Ans:  $\frac{\pi}{4}$ ,  $\left( \frac{7}{5}, \frac{4}{5} \right)$  ]

- (13) If the intercepts on the axes by the line  $x \cos \alpha + y \sin \alpha = p$  are  $a$  and  $b$ , prove that  $a^{-2} + b^{-2} = p^{-2}$ .

- (14) Given  $A(2, 2)$ ,  $B(0, 4)$  and  $C(3, 3)$ , find the equation of (i) the median of triangle ABC through A, (ii) the altitude of triangle ABC through A (iii) the perpendicular bisector of  $\overline{BC}$  and (iv) the bisector of  $\angle BAC$ .

[ Ans: (i)  $3x + y = 8$ , (ii)  $3x - y = 4$ , (iii)  $3x - y = 1$  and (iv)  $x = 2$  ]

- (15) Equations of the two of the sides of a parallelogram are  $3x - y - 2 = 0$  and  $x - y - 1 = 0$  and one of its vertices is  $(2, 3)$ . Find the equations of the remaining sides.

[ Ans:  $x - y + 1 = 0$ ,  $3x - y - 3 = 0$  ]

(16) A line passes through  $(\sqrt{3}, -1)$  and the length of the segment perpendicular to it from the origin is  $\sqrt{2}$ . Find the equation of the line.

[Ans:  $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 4$ ,  $(\sqrt{3} - 1)x - (\sqrt{3} + 1)y = 4$ ]

(17) Find the equation of a line through  $(2, 6)$  if the length of the perpendicular segment to it from the origin is 2.

[Ans:  $x = 2$ ,  $4x - 3y + 10 = 0$ ]

(18) Find the equation of the line which passes through  $(3, -2)$  and which makes an angle of  $60^\circ$  with the line  $\sqrt{3}x + y = 1$ .

[Ans:  $y + 2 = 0$ ,  $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$ ]

(19) Find the equation of the line which passes through  $(3, 4)$  and which makes an angle of  $45^\circ$  with the line  $3x + 4y = 2$ .

[Ans:  $x - 7y + 25 = 0$ ,  $7x + y - 25 = 0$ ]

(20) Find the points on  $2x - y = 1$  which are at a distance  $\sqrt{5}$  from  $(1, -1)$ .

[Ans:  $(0, 1)$ ,  $(2, -3)$ ]

(21) Find the points on  $2x + y = 1$  which are at a distance 2 from  $(1, 1)$ .

[Ans:  $(1, -1)$ ,  $\left(-\frac{3}{5}, \frac{11}{5}\right)$ ]

(22) A line intersects X- and Y-axes at A and B respectively. If  $AB = 10$  and  $3OA = 4OB$ , then find the equation of the line.

(Ans:  $\pm 3x \pm 4y = 24$ )

(23) The points  $(1, 2)$  and  $(3, 8)$  are a pair of opposite vertices of a square. Find the equations of the lines containing its sides and diagonals.

(Ans:  $x - 2y + 3 = 0$ ,  $x - 2y + 13 = 0$ ,  $2x + y - 4 = 0$ ,  $2x + y - 14 = 0$ ,  
 $3x - y - 1 = 0$ ,  $x + 3y - 17 = 0$ )

(24) An adjacent pair of vertices of a square is  $(-1, 3)$  and  $(2, -1)$ . Find the remaining vertices.

[Ans:  $(6, 2)$ ,  $(3, 6)$ ,  $(-2, -4)$ ,  $(-5, 0)$ ]

(25) Find the equations of the lines passing through  $(-2, 3)$  which form an equilateral triangle with the line  $\sqrt{3}x - 3y + 16 = 0$

(Ans:  $x + 2 = 0$ ,  $x + \sqrt{3}y + 2 - 3\sqrt{3} = 0$ )

(26) Area of triangle ABC is 4. The co-ordinates of A and B are  $(2, 1)$  and  $(4, 3)$ . Find the co-ordinates of C if it lies on the line  $3x - y - 1 = 0$

[Ans:  $(2, 5)$ ,  $(-2, -7)$ ]

(27) Find the equation of a line passing through  $(2, 3)$  and which contains a segment of length 2 between the lines  $2x + y - 5 = 0$  and  $2x + y - 3 = 0$ .

[Ans:  $3x + 4y - 18 = 0$ ,  $x = 2$ ]

(28) Show that the centroid of the triangle ABC lies on the line  $21x + 27y - 74 = 0$  if C lies on  $7x + 9y - 10 = 0$  and A and B have co-ordinates  $(6, 3)$  and  $(-2, 1)$ .

(29) If  $\frac{1}{a} + \frac{1}{b} = k$ , then prove that all lines  $\frac{x}{a} + \frac{y}{b} = 1$  pass through a fixed point.

(30) Prove that if  $a + b + c = 0$ , and  $b^2 \neq ac$ ,  $c^2 \neq ab$  and  $a^2 \neq bc$ , then the lines  $ax + by + c = 0$ ,  $bx + cy + a = 0$  and  $cx + ay + b = 0$  are concurrent and find their point of concurrence.

[Ans:  $(1, 1)$ ]

(31) Find the co-ordinates of the foot of the perpendicular from the origin on the line

$$\frac{x}{a} + \frac{y}{b} = 1$$

[Ans:  $\left( \frac{ab^2}{a^2 + b^2}, \frac{a^2b}{a^2 + b^2} \right)$ ]

(32) Without finding the point of intersection of the lines  $x - 2y - 2 = 0$  and  $2x - 5y + 1 = 0$ , find the equation of the line passing through that point and satisfying the given conditions:

- (i) whose both intercepts are equal,
- (ii) whose sum of the two intercepts is zero, but their product is not zero,
- (iii) whose distance from origin is 13 units and
- (iv) for which the product of both intercepts is -30.

[Ans: (i)  $x + y - 17 = 0$ ,  $5x = 12y$ , (ii)  $x - y - 7 = 0$ , (iii)  $12x + 5y - 169 = 0$ , (iv)  $5x - 6y - 30 = 0$ ,  $5x - 24y + 60 = 0$ ]

(33) Find the points on the line  $3x - 2y - 2 = 0$  which are at a distance 3 units from  $3x + 4y - 8 = 0$ .

[Ans:  $\left(-\frac{1}{3}, -\frac{3}{2}\right)$ ,  $\left(3, \frac{7}{2}\right)$ ]

(34) Prove that  $x^2 - y^2 - 2xy \tan \theta + 2ay \sec \theta - a^2 = 0$  represents a pair of lines and find their point of intersection.

[Ans:  $(a \sin \theta, a \cos \theta)$ ]

(35) Find the measure of the angle between the lines

$$(\cos^2 \alpha - \cos^2 \theta) x^2 - 2xy \sin 2\theta + (\cos^2 \theta - \sin^2 \alpha) y^2 = 0, \quad 0 < \alpha < \frac{\pi}{4}.$$

[Ans:  $2\alpha$ ]

(36) Prove that the difference between the slopes of the lines

$$(\tan^2 \theta + \cos^2 \theta) x^2 - 2xy \tan \theta + y^2 \sin^2 \theta = 0 \text{ is } 2.$$

(37) Prove that the equation of the lines through the origin which makes an angle of measure  $\alpha$  with  $x + y = 0$  is  $x^2 + 2xy \sec 2\alpha + y^2 = 0$  ( $0 < \alpha < \frac{\pi}{4}$ ).

(38) If the equation  $x^2 - \lambda xy + 4y^2 + x + 2y - 2 = 0$  represents two lines, then find  $\lambda$ .

[Ans: -4, 5]

- (39) The sides of a triangle are along the lines  $x - 2y + 2 = 0$ ,  $3x - y + 6 = 0$  and  $x - y = 0$ . Find the orthocentre of the triangle.

[ Ans:  $(-7, 5)$  ]

- (40) Find the area of the triangle whose sides are along the lines  $x = 0$ ,  $y = m_1x + c_1$  and  $y = m_2x + c_2$ .

[ Ans:  $\frac{(c_1 - c_2)^2}{2 |m_1 - m_2|}$  ]

- (41) Find the area of the parallelogram whose sides are along the lines  $y = mx + a$ ,  $y = mx + b$ ,  $y = nx + c$  and  $y = nx + d$ .

[ Ans:  $\left| \frac{(a - b)(c - d)}{m - n} \right|$  ]

- (42) Find the points on the line  $x - 5y - 13 = 0$  which are at a distance of 2 units from the line  $12x - 5y + 26 = 0$ .

[ Ans:  $\left( 1, \frac{12}{5} \right), \left( -3, \frac{16}{5} \right)$  ]

- (43) One pair of opposite vertices of a rhombus is  $(-2, 5)$  and  $(6, 7)$ . One of its sides is along the line  $x - 2y + 12 = 0$ . Find the equations of the lines along which the remaining sides and diagonals of the rhombus lie.

[ Ans:  $x - 2y + 8 = 0$ ,  $x - 38y + 260 = 0$ ,  $x - 38y + 192 = 0$ ,  $4x + y - 14 = 0$  and  $x - 4y + 22 = 0$  ]

- (44) In triangle ABC, A is  $(3, 4)$  and the lines containing two of the altitudes are  $4x + y = 0$  and  $3x - 4y + 23 = 0$ . Find the co-ordinates of B and C.

[ Ans:  $(-5, 2), (-3, 12)$  ]

- (45) Find the co-ordinates of the foot of the perpendicular from the point  $(a, 0)$  on the line  $y = mx + \frac{a}{m}$ ,  $m \neq 0$ .

[ Ans:  $(0, a/m)$  ]

- (46) Co-ordinates of A in triangle ABC are (1, -2) and the equations of the perpendicular bisectors of  $\overline{AB}$  and  $\overline{AC}$  are  $x - y + 5 = 0$  and  $x + 2y = 0$ . Find the co-ordinates of B and C.

$$\left[ \text{Ans: } (-7, 6), \left( \frac{11}{5}, \frac{2}{5} \right) \right]$$

- (47) In triangle ABC, A is (4, -3) and two of the medians lie along the lines  $2x + y + 1 = 0$  and  $x + 5y - 1 = 0$ . Find the co-ordinates of B and C.

$$[\text{Ans: } (-2, 3), (-4, 1)]$$

- (48) Find the combined equation of the lines through the origin which are perpendicular to the lines  $ax^2 + 2hxy + by^2 = 0$ .

$$[\text{Ans: } bx^2 - 2hxy + ay^2 = 0]$$

- (49) If the line connecting A ( $at_1^2, 2at_1$ ) and B ( $at_2^2, 2at_2$ ) passes through the point (a, 0), then prove that  $t_1 t_2 = -1$ .

- (50) Prove that the equation of the line passing through A ( $a \cos \alpha, b \sin \alpha$ ) and B ( $a \cos \beta, b \sin \beta$ ) is  $\frac{x}{a} \cos \frac{\alpha + \beta}{2} + \frac{y}{b} \sin \frac{\alpha + \beta}{2} = \cos \frac{\alpha - \beta}{2}$ .

- (51) Show that the equation of the line passing through ( $a \cos^3 \theta, a \sin^3 \theta$ ) and perpendicular to  $x \sec \theta + y \operatorname{cosec} \theta = a$  is  $x \cos \theta - y \sin \theta = a \cos 2\theta$ .

- (52) Find the equation of the line passing through the origin and cutting off a segment of length  $\sqrt{10}$  between the lines  $2x - y + 1 = 0$  and  $2x - y + 6 = 0$ .

$$[\text{Ans: } x - 3y = 0, 3x + y = 0]$$

- (53) If p and p' are the perpendicular distances of the origin from the lines  $x \sec \theta + y \operatorname{cosec} \theta = a$  and  $x \cos \theta - y \sin \theta = a \cos 2\theta$  respectively, then prove that  $4p^2 + p'^2 = a^2$ .

- (54) If a perpendicular from origin on a line passing through the point of intersection of  $4x - y - 2 = 0$  and  $2x + y - 10 = 0$  is of length 2, then find the equation of the line.

[ Ans:  $x = 2, 4x - 3y + 10 = 0$  ]

- (55) Find the orthocentre of the triangle ABC formed by the three lines  $y = a(x - b - c)$ ,  $y = b(x - c - a)$  and  $y = c(x - a - b)$ .

[ Ans:  $(-abc, 1)$  ]

- (56) If the line  $\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta}$  passing through  $A(x_1, y_1)$  meets the line  $ax + by + c = 0$

at B, then prove that  $AB = \frac{|ax_1 + by_1 + c|}{a \cos \theta + b \sin \theta}$ .

- (57) If the line  $lx + my + n = 0$  is the perpendicular bisector of  $\overline{AB}$  joining  $A(x_1, y_1)$  and  $B(x_2, y_2)$ , then show that  $\frac{x_1 - x_2}{l} = \frac{y_1 - y_2}{m} = \frac{2(lx_1 + my_1 + n)}{l^2 + m^2}$ .

- (58) If in a pair of straight lines represented by the equation  $ax^2 + 2hxy + by^2 = 0$ , the slope of one line is  $k$  times that of the other, then prove that  $4kh^2 = ab(1 + k)^2$ .

- (59) The line  $3x + 2y = 24$  meets the Y-axis at A and the X-axis at B. The perpendicular bisector of  $\overline{AB}$  meets the line through  $(0, -1)$  parallel to X-axis at C. Find the area of the triangle ABC.

[ Ans: 91 sq. units ]

- (60) A line  $4x + y = 1$  through the point  $A(2, -7)$  meets the line  $\overleftrightarrow{BC}$  whose equation is  $3x - 4y + 1 = 0$  at the point B. Find the equation of the line  $\overleftrightarrow{AC}$ , so that  $AB = AC$ .

[ Ans:  $52x + 89y + 519 = 0$  ]

- (61) Find orthocentre of the triangle whose vertices are  $[at_1t_2, a(t_1 + t_2)]$ ,  $[at_2t_3, a(t_2 + t_3)]$  and  $[at_3t_1, a(t_3 + t_1)]$ .

[ Ans:  $[-a, a(t_1 + t_2 + t_3 + t_1 t_2 t_3)]$  ]



(62) Show that if  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$  are the vertices of a triangle, then

(i) the equation of the median through  $A$  is given by

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \quad \text{and}$$

(ii) the equation of the internal bisector of angle  $A$  is given by

$$b \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0, \quad \text{where } b = AC \text{ and } c = AB.$$

(63) Lines  $L_1 \equiv ax + by + c = 0$  and  $L_2 \equiv lx + my + n = 0$  intersect at the point  $P$  and make an angle  $\theta$  with each other. Find the equation of the line different from  $L_2$  which passes through  $P$  and makes the same angle  $\theta$  with  $L_1$ .

[Ans:  $2(a^2 + b^2)(ax + by + c) - (a^2 - b^2)(lx + my + n) = 0$ ]

(64) Find the locus of the mid-point of the portion of the variable line  $x \cos \alpha + y \sin \alpha = p$ , intercepted by the co-ordinate axes, given that  $p$  remains constant.

[Ans:  $x^{-2} + y^{-2} = 4p^{-2}$ ]

(65) A variable straight line drawn through the point of intersection of the lines  $bx + ay = ab$  and  $ax + by = ab$  meets the co-ordinate axes in  $A$  and  $B$ . Show that the locus of the mid-point of  $\overline{AB}$  is the curve  $2xy(a + b) = ab(x + y)$ .

(66) Show that the quadrilateral formed by the lines  $ax \pm by + c = 0$  and  $ax \pm by - c = 0$  is a rhombus and that its area is  $\frac{2c^2}{|ab|}$ .

(67) Suppose  $a \neq b$ ,  $b \neq c$ ,  $c \neq a$ ,  $a \neq 1$ ,  $b \neq 1$ ,  $c \neq 1$  and the lines  $ax + y + 1 = 0$ ,  $x + by + 1 = 0$  and  $x + y + c = 0$  are concurrent, then prove that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1.$$

(68) The equation of a line bisecting an angle between two lines is  $2x + 3y - 1 = 0$ . If one of the two lines has equation  $x + 2y = 1$ , find the equation of the other line.

[Ans:  $19x + 22y = 3$ ]

(69) Prove that all the chords of the curve  $3x^2 - y^2 - 2x + 4y = 0$  subtending right angle at the origin are concurrent.

(70) A line cuts X-axis at  $A(7, 0)$  and the Y-axis at  $B(0, -5)$ . A variable line PQ is drawn perpendicular to AB cutting the X-axis at P and the Y-axis at Q. If AQ and BP intersect in R, find the locus of R. [IIT 1990]

[Ans:  $x^2 + y^2 - 7x + 5y = 0$ ]

(71) Straight lines  $3x + 4y = 5$  and  $4x - y = 15$  intersect at the point A. Points B and C are chosen on these two lines such that  $AB = AC$ . Determine the possible equations of the line BC passing through the point  $(1, 2)$ . [IIT 1990]

[Ans:  $7x + y - 9 = 0$ ,  $x - 7y + 13 = 0$ ]

(72) The sides of a triangle are along the lines  $L_i \equiv x \cos \alpha_i + y \sin \alpha_i - p_i = 0$ ,  $i = 1, 2, 3$ . Prove that the orthocentre of the triangle is given by

$L_1 \cos(\alpha_2 - \alpha_3) = L_2 \cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2)$ .

(73) A line through  $A(-2, -3)$  meets the lines  $x + 3y - 9 = 0$  and  $x + y + 1 = 0$  at B and C respectively such that  $AB \cdot AC = 20$ . Find the equation of  $\overleftrightarrow{AB}$ .

(74) A line through  $A(-5, -4)$  meets the lines  $x + 3y + 2 = 0$ ,  $2x + y + 4 = 0$  and  $x - y - 5 = 0$  at B, C, and D respectively. If AC, AB, AD are in H.P., then find the equation of  $\overleftrightarrow{AB}$ .

(75) Find the equation of the line passing through the point of intersection of the lines  $x + y = 2$  and  $3x - y = 2$  and for which perpendicular distance from the origin is the shortest.