## A FILL IN THE BLANKS

1. Let $a, b, c$ be positive real numbers

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}.$$

Then $\tan\theta$ equals ... **(IIT 1981; 2M)**

2. The numerical value of $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$ is equal to... **(IIT 1984; 2M)**

3. The greater of the two angles $A = 2\tan^{-1}(2\sqrt{2}-1)$ and $B = 3\sin^{-1}(1/3) + \sin^{-1}(3/5)$ is ... **(IIT 1989; 2M)**

## C OBJECTIVE QUESTIONS

➥ ***Only one option is correct :***

1. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is : **(IIT 1983; 1M)**

(a) $\frac{6}{17}$ (b) $\frac{17}{6}$

(c) $\frac{16}{7}$ (d) none of these

2. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is : **(IIT 1986; 2M)**

(a) $-\frac{2\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $\frac{5\pi}{3}$

(e) none of these.

3. The number of real solutions of

$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is: **(IIT 1999; 2M)**

(a) zero (b) one

(c) two (d) infinite

4. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\right) = \frac{\pi}{2}$, for $0 < |x| < \sqrt{2}$, then $x$ equals : **(IIT 2001)**

(a) 1/2 (b) 1

(c) −1/2 (d) −1

5. The value of $x$ for which $\sin(\cot^{-1}(1-x)) = \cos(\tan^{-1}x)$ is : **(IIT 2004)**

(a) $\frac{1}{2}$ (b) 1

(c) 0 (d) $-\frac{1}{2}$

## E SUBJECTIVE QUESTIONS

1. Find the value of :

$\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$, where $0 \le \cos^{-1}x \le \pi$ and $-\pi/2 \le \sin^{-1}x \le \pi/2$. **(IIT 1981; 2M)**

2. Prove that $\cos\tan^{-1}\{(\sin\cot^{-1}x)\} = \sqrt{\frac{x^2+1}{x^2+2}}$. **(IIT 2002; 5M)**

## H Match The Column

1. Let $(x, y)$ be such that $\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy)=\dfrac{\pi}{2}$. **(IIT 2007)**

Match the statements in **Column I** with the values in **Column II**.

| | Column I | | Column II |
|---|---|---|---|
| (A) | If $a=1$ and $b=0$, then $(x, y)$ | (p) | lies on the circle $x^2+y^2=1$ |
| (B) | If $a=1$ and $b=1$, then $(x, y)$ | (q) | lies on $(x^2-1)(y^2-1)=0$ |
| (C) | If $a=1$ and $b=2$, then $(x, y)$ | (r) | lies on $y=x$ |
| (D) | If $a=2$ and $b=2$, then $(x, y)$ | (s) | lies on $(4x^2-1)(y^2-1)=0$ |

# ANSWERS

### A Fill in the blanks

**1.** 0 **2.** $-\dfrac{7}{17}$ **3.** $A>B$

### C Objective Questions (Only one option)

**1.** b **2.** c **3.** c **4.** b **5.** d

### E Subjective Question

**1.** $\dfrac{2\sqrt{6}}{5}$

### H Match the Column

**1.** A—p, B—q, C—p, D—s

# SOLUTIONS

### A Fill in the Blanks

**1.** We have

$$\theta=\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ac}}+\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$$

$$=\tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

Where, $x=\sqrt{\dfrac{a(a+b+c)}{bc}}$, $y=\sqrt{\dfrac{b(a+b+c)}{ac}}$

and $z=\sqrt{\dfrac{c(a+b+c)}{ab}}$

$$=\tan^{-1}\left(\frac{\sqrt{a+b+c}\left(\sqrt{\frac{a}{bc}}+\sqrt{\frac{b}{ca}}+\sqrt{\frac{c}{ab}}\right)-(a+b+c)\sqrt{\frac{a+b+c}{abc}}}{1-(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{\frac{a+b+c}{abc}}(a+b+c)-(a+b+c)\sqrt{\frac{a+b+c}{abc}}}{1-\frac{(a+b+c)(ab+bc+ca)}{abc}}\right)$$

$$\Rightarrow \theta=\tan^{-1}0$$

$$\Rightarrow \tan\theta=0$$

**2.** $$\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right\}=\tan\left\{\tan^{-1}\left(\frac{2\cdot\frac{1}{5}}{1-\frac{1}{25}}\right)-\frac{\pi}{4}\right\}$$

$$=\tan\left\{\tan^{-1}\left(\frac{5}{12}\right)-\frac{\pi}{4}\right\}$$

$$=\frac{\tan\left(\tan^{-1}\left(\frac{5}{12}\right)\right)-\tan\left(\frac{\pi}{4}\right)}{1+\tan\left(\tan^{-1}\frac{5}{12}\right)\tan\frac{\pi}{4}}$$

$$=\frac{\frac{5}{12}-1}{1+\frac{5}{12}\cdot 1}=\frac{-7}{17}$$

**3.** $A=2\tan^{-1}(2\sqrt{2}-1)$ and $B=3\sin^{-1}\left(\dfrac{1}{3}\right)+\sin^{-1}\left(\dfrac{3}{5}\right)$

Here, $A=2\tan^{-1}(2\sqrt{2}-1)$

$=2\tan^{-1}(2\times 1.414-1)=2\tan^{-1}(1.828)$