# Structure of Atom

## Q 1.

Naturally occurring consists of two isotopes whose atomic weights are 10.01 and 11.01. The atomic weight of natural boron is 10.81. Calculate the percentage of each isotope in natural boron. **(IIT JEE 1978)**

## Q 2.

Account for the following. Limit your answer to two sentences: **(IIT JEE 1979)**

## Q 3.

The energy of the electron in the second and the third Bohr's orbits of the hydrogen atom is $-5.42 * 10^{-12}$ erg and $-2.41 * 10^{-12}$ erg respectively. Calculate the wavelength of the emitted radiation when the electron drops from the third to the second orbit.

**(IIT JEE 1981 – 3 Marks)**

## Q 4.

Calculate the wavelength in Angstrom of the photon that is emitted when an electron in the Bohr orbit, n = 2 returns to the orbit, n = 1 in the hydrogen atom. The ionization potential of the ground state hydrogen atom is $2.17 * 10^{-11}$ erg per atom. **(IIT JEE 1982 – 4 Marks)**

## Q 5.

The electron energy in hydrogen atom is given by $E = (-2.17 * 10^{-12})/n^2$ ergs. Calculate the energy required to remove an electron completely from the n = 2 orbit. What is the longest wavelength (in cm) of light that can be used to cause this transition?

**(IIT JEE 1984 – 3 Marks)**

## Q 6.

Give reasons why the ground state outermost electronic configuration of silicon is:

2s [↑↓] 3p [↑ | ↑ | ] and not 3s [↑↓] 3p [↑ | ↓ | ]

**(IIT JEE 1985 – 2 Marks)**

## Q 7.

What is the maximum number of electrons that may be present in all the atomic orbital with principal quantum number 3 and azimuthally quantum number 2? **(IIT JEE 1985 – 2 Marks)**

## Q 8.

According to Bohr's theory, the electronic energy of hydrogen atom in the nth Bohr's orbit is given by $E_n = -21 * 10^{-19}/n^{-2}$ J. Calculate the longest wavelength of light that will be needed to remove an electron from the third Bohr orbit of the $He^+$ ion. **(IIT JEE 1990 – 3 Marks)**

## Q 9.

Estimate the difference in energy between 1st and 2nd Bohr orbit for a hydrogen atom. At what minimum atomic number, a transition from n = 2 to n = 1 energy level would result in the emission of X-rays with $\lambda = 3.0 * 10^{-8}$m? Which hydrogen atom – like species does this atomic number correspond to? **(IIT JEE 1993 – 5 Marks)**

## Q 10.

What transition in the hydrogen spectrum would have the same wavelength as the Balmer transition n = 4 to n = 2 of $He^+$ spectrum? **(IIT JEE 1993 – 3 Marks)**

## Q 11.

Find out the number of waves made by a Bohr electron in one complete revolution in its 3rd orbit. **(IIT JEE 1994 – 3 Marks)**

## Q 12.

Iodine molecule dissociates into atoms after absorbing light of 4500 Å. If one quantum of radiation is absorbed by each molecule, calculate the kinetic energy of iodine atoms. (Bond energy of $I_2$ = 240 kJ $mol^{-1}$) **(IIT JEE 1995 – 2 Marks)**

## Q 13.

Calculate the wave number for the shortest wavelength transition in the Balmer series of atomic hydrogen. **(IIT JEE 1996 – 1 Marks)**

## Q 14.

Consider the hydrogen atom to be a proton embedded in a cavity of radius $a_o$ (Bohr radius) whose charge in neutralized by the addition of an electron to the cavity in vacuum, infinitely slowly. Estimate the average total energy of an electron in its ground state in a hydr5ogen atom as the work done in the above neutralization process. Also, if the magnitude of the average kinetic energy is half the magnitude of the average potential energy, find the average potential energy. **(IIT JEE 1996 – 2 Marks)**

## Q 15.

With what velocity should a ∝-particle travel towards the nucleus of a copper atom so as to arrive at a distance $10^{-13}$ metre from the nucleus of the copper atom?

**(IIT JEE 1997C – 3 Marks)**

## Q 16.

An electron beam can undergo diffraction by crystals. Through what potential should a beam of electrons be accelerated so that its wavelength becomes equal to 1.54 Å?

**(IIT JEE 1997 – 2 Marks)**

## Q 17.

Calculate the energy required to excite on litre of hydrogen gas at 1 atm and 298 K to the first excited state of atomic hydrogen. The energy for the dissociation of H-H bond is 436 kJ $mol^1$.

**(IIT JEE 2000 – 4 Marks)**

## Q 18.

Wavelength of high energy transition of H-atoms is 91.2nm. Calculate the corresponding wavelength of he atoms. **(IIT JEE 2003 – 2 Marks)**

## Q 19.

The Schrodinger wave equation for hydrogen atom is $\Psi_{2s} = \frac{1}{4\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r_0}{a_0}\right)e^{-r_0/a_0}$

Where $a_0$ is Bohr's radius. If the radial node in 2s be at $r_0$, then find $r_0$ in terms of $a_0$.

**(IIT JEE 2004 – 1 Marks)**

## Q 20.

A ball of mass 100 g is moving with 100 $ms^{-1}$. Find its wavelength. Find out atomic number and mass number of the element X. Also identify the element. **(IIT JEE 2004 – 1 Marks)**

## Q 21.

Find the velocity ($ms^{-1}$) of electron in first Bohr's orbit of radius $a_0$. Also find the de Broglie's wavelength (in m). Find the orbital angular momentum of 2P orbital of hydrogen atom in units of units of $h/2\pi$. **(IIT JEE 2005 – 2 Marks)**

# Structure of Atom

## SOL 1.

Let the % of isotope with At. wt. 10.01 = x

∴ % of isotope with At. wt. 11.01 = (100 - x)

At. wt. of boron = x *10.01 + (100 - x) *11.01/100

=> 10.81 = x *10.01 + (100 - x) *11.01/100

∴ x = 20

**Hence** % of isotope with At. wt. 10.01 = 20%

∴ % of isotope with At. wt. 11.01 = 100 – 20 = **80%**.

## SOL 2.

Elements usually have one or more isotopes and these isotopes have different atomic masses. So atomic weights are fractional number.

## SOL 3.

**TIPS/Formulae:** $\Delta E = E_3 - E_2 = hv = hc/\lambda$ or $\lambda = hc/E_3 - E_2$

Given $E_2 = - 5.42 * 10^{-12}$ erg

$E_3 = - 2.41 * 10^{-12}$ erg

∴ $6.626 *10^{-27} *3 *10^{10}/-2.41 *10^{-12} - (-5.42 * 10^{-12}) = 19.878 *10^{-17}/3.01 *10^{-12} = 6.604 * 10^{-5}$ cm = **6.604 Å**

## SOL 4.

**TIPS/Formulae:** (i) Energy of $n^{th}$ orbit $E_n = E_1/n^2$

(ii) Difference in energy = $E_1 - E_2 = hv = hc/\lambda$ or $\lambda = hc/E_1 - E_2$

Given $E_1 = 2.17 * 10^{-11}$

∴ Energy of second orbit = $E_2 = 2.17 *10^{-11}/2^2 = 0.5425 *10^{-11}$ erg

$\Delta = E_1 - E_2 = 2.17 * 10^{-11} - 0.5425 * 10^{-11} = 1.0275 * 10^{-11}$ erg

$\lambda = 6.62 *10^{-27} *3 *10^{10}/1.6275 *10^{-11} = 12.20 * 10^{-6}$ cm = **1220 Å**

## SOL 5.

To calculate the energy required to remove electron from atom, $n = \infty$ is to be taken.

Energy of an electron in nth orbit of hydrogen is given by

$E = 21.7 * 10^{-12} * 1/n^2$ ergs

∴ $\Delta E = 21.7 * 10^{-12} (1/2^2 - 1/\infty^2)$

$= - 21.7 * 10^{-12} (1/4 - 0) = 21.7 * 10^{-12} * 1/4$

**$= - 5.42 * 10^{-12}$ ergs**

∴ $\Delta E = hc/\lambda$ (∵ $v = c/\lambda$)

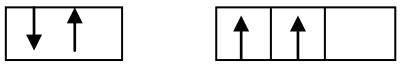
Or $\lambda = hc/\Delta E$

Substituting the values, $\lambda = 6.627 *10^{-27} *3 *10^{10}/5.42 * 10^{-12}$

**$= 3.67 * 10^{-5}$ cm**

## SOL 6.

Ground state electronic configuration of Si|



3s $3p_x$ $3p_y$ $3p_z$

is in accordance with Hund's rule which states that electron pairing in any orbital (s, p, d or f) cannot take place until each orbital of the same sub-level contain 1 electron each of like spin.

## SOL 7.

For n = 3 and l = 2 (i.e., 3d orbital), the values of m varies from -2 to +2, -1, 0, +1, +2 and for each 'm' there are 2 values of 's', i.e. +½ and –½ .

∴ Maximum no. of electrons in all the five d-orbitals is **10**.

## SOL 8.

$E_n$ of H = $-21.76 * 10^{-19}/n^2$ J

∴ $E_n$ of $He^+$ = $-21.76 * 10^{-19}/n^2 * Z^2$ J

∴ $E_3$ of $He^+$ = $-21.76 * 10^{-19} * 4/9$ J

Hence energy equivalent to $E_3$ must be supplied to remove the electron from 3rd orbit of $He^+$. Wavelength corresponding to this energy can be determined by applying the relation.

E = hc/λ

or $\lambda = hc/E = 6.625 * 10^{-34} * 3 */ 10^8 * 9/21.76 * 10^{-19} * 4 = 2055 * 10^{-10}$ m **= 2055 Å**

## SOL 9.

**TIPS/Formulae:** $\Delta E = RhcZ^2 (1/n_1^2 - 1/n_2^2)$

Here, $R = 1.0967 * 10^7 \ m^{-1}$

$h = 6.626 * 10^{-34}$ J sec, $c = 3 * 10^8$ m/sec

$n_1 = 1$, $n_2 = 2$ and for H-atom, Z = 1

$E_2 - E_1 = 1.0967 * 10^7 * 6.626 * 10^{-34} * 3 * 10^8 (1/1 - 1/4)$

$\Delta E = 1.0967 * 6.626 * 3 * ¾ * 10^{-19}$ J

$= 16.3512 * 10^{-19}$ J

$= 16.3512 * 10^{-19}/1.6 * 10^{-19}$ eV = **10.22 eV**

$\Delta E = hc/\lambda = RhcZ^2 (1/n_1^2 - 1/n_2^2)$

$1/\lambda = RZ^2 (1/1 - 1/4) = RZ^2 * 3/4$

Given, $\lambda = 3 * 10^{-8 =}$ m

∴ $1/3 * 10^{-8} = 1.0967 = Z^2 * 3/4 * 10^7$

∴ $Z^2 = 10^8 * 4/3 * 3 * 1.0967 * 10^7 = 40/9 * 1.0967 = 4$

∴ Z = 2

So it corresponds to $He^+$ which has 1 electron like hydrogen.

**SOL10.**

For $He^+$ ion, we have

$1/\lambda = Z^2R_H [1/n_1^2 - 1/n_2^2]$

$= (2)^2R_H [1/(2)^2 - 1/(4)^2] = R_H\, 3/4$ ....(i)

Now for hydrogen atom $1/\lambda = R_H [1/n_1^2 - 1/n_2^2]$ ....(ii)

Equating equation (i) and (ii), we get

$1/n_1^2 - 1/n_2^2 = 3/4$

Obviously, $n_1 = 1$ and $n_2 = 2$

Hence, the transition **n = 2 to n = 1** in hydrogen atom will have the same wavelength as the transition, n = 4 to n = 2 in $He^+$ species.

**SOL 11.**

**TIPS/Formulae:** Number of waves = n(n - 1)/2 where n = Principal quantum number or number of orbit number of waves = 3(3 - 1)/2 = 3 * 2/2 = 3

**ALTERNATIVE SOLUTIONS:**

In general, the number of waves made by a Bohr electron in an orbit is equal to its quantum number.

According to Bohr's postulate of angular momentum, in the 3rd orbit

$Mur = n\, h/2\pi$

$Mur = 3\, (h/2\pi)$ .....(i) [n = 3]

According to de Broglie relationship

$\lambda = h/mu$ ....(ii)

Substituting (ii) in (i), we get

$(h/\lambda)\, r = 3\, (h/2\pi)$ or $3\lambda = 2\pi r$

$[\because mu = h/\lambda]$

Thus the circumference of the 3rd orbit is equal to 3 times the wavelength of electron i.e. the electron makes **three** revolution around the 3rd orbit.

**ALTERNATIVE SOLUTION:**

$r_n$ for H = $r_1 * n^2$

$r_3$ for H = $0.529 * 9 * 10^{-8}$ cm

$= .529 * 9 * 10^{-10}$ m $(\because r_1 = 0.529$ Å$)$

Also $u_n = Z\, u_1/n$ ;

$\therefore u_3 = 2.19 * 10^8/3$ cm sec$^{-1}$ = 2.19 *10+16/3 m sec$^{-1}$

$(\because u_1 = 2.19 * 10^8$ cm sec$^{-1})$

$\therefore$ No. of waves in one round

$= 2\pi\, r_3/\lambda = 2\pi\, r_3/h/mv_3 = 2\pi\, r_3 * m/h$

$= 2 * 22 * 0.529 * 9 * 10^{-10} * 2.19 * 10^6 * 9.108 * 10^{-31}/7 * 3 * 6.62 * 10^{-34}$ **= 3**

**SOL 12.**

Bond energy of $I_2$ 240kJ mol$^{-1}$ = $240 * 10^3$ J mol$^{-1}$

$= 240 * 10^3/6.023 * 10^{23}$ J molecule$^{-1}$

$= 3.984 * 10^{-19}$ J molecule$^{-1}$

Energy absorbed = $hc/\lambda = 6.626 * 10^{-34}$ Js $* 3 * 10^8$ ms$^{-1}/4500 * 10^{-10}$ m

$= 4.417 * 10^{-19}$ J
Kinetic energy = Absorbed energy – Bond energy
∴ Kinetic energy = $4.417 *10^{-19} – 3.984 *10^{-19}$ J
$= 4.33 * 10^{-20}$ J
∴ Kinetic energy of each atom of iodine
$= 4.33 *10^{-20}/2 =$ **$2.165 *10^{-20}$**

## SOL 13.

The shortest wavelength transition in the Barlmer series corresponds to the transition $n = 2 \rightarrow n = \infty$. Hence, $n_1 = 2$, $n_2 = \infty$ Balmer
$\bar{\upsilon} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = (109677\ cm^{-1})\ (1/2^2 – 1/\infty^2) =$ **$27419.25\ cm^{-1}$**

## SOL 14.

Work done while bringing an electron infinitely slowly from infinity to proton of radius $a_0$ is given as follows
$W = - e^2/4\pi\varepsilon_0.a_0$
**NOTE :** This work done is equal to the total energy of an electron in its ground state in the hydrogen atom. At this stage, the electron is not moving and do not possess any K.E., so this total energy is equal to the potential energy.
T.E. = P.E + K.E. = $e^2/4\pi\varepsilon_0.a_0$ ....(i)
In order the electron to be captured by proton to form a ground state hydrogen atom it should also attain
K.E. = $e^2/8\pi\varepsilon_0 a_0$
(it is given that magnitude of K.E. is half the magnitude of P.E. Note that P.E. is –ve and K.E is +ve)
∴ T.E. = P.E + K.E. = $- e^2/4\pi\varepsilon_0 a_0 + e^2/8\pi\varepsilon_0 a_0$
or T.E. = $- e^2/8\pi\varepsilon_0 a_0$
P.E. = 2 * T.E. = 2 * $-e^2/8\pi \in_0 a_0$ or P.E. = **$-e^2/8\pi \in_0 a_0$**

## SOL 15.

As the ∝-particle travelling with velocity, 'u', stops at a distance $10^{-13}$ m, its K.E. becomes zero and gets converted into P.E.
∴ $1/2\ mu^2 = 1/4\pi\ \varepsilon_0 * 2Ze^2/r$ or $u^2 = Ze^2/4\pi\varepsilon_0\ m.r$
Here, Z = 29 for Cu atom
=> $u^2\ 29 *(1.6 *10^{-19})^2/3.14 *8.85 *10^{-12} *(4 *1.672 *10^{-27}) *10^{-13}$
∴ **$u = 6.3 * 10^6$ m $sec^{-1}$**

## SOL 16.

$1/2\ mu^2 = eV$; also $\lambda = h/mu$ or $u = h/m\lambda$
∴ $1/2\ m\ h^2/m^2\ \lambda^2 = eV$ or $V = 1/2\ \ h^2/m\lambda^2\ e$
$V = 1 *(6.62 *10^{-34})^2/2 *9.108 * 10^{-31} *(1.54 *10^{-10})^2 *1.602 * 10^{-19} =$ **63.3 volt**

**SOL 17.**

Determination of number of moles of hydrogen gas,

N = PV/RT = 1 *1/0.082 *298 = 0.0409

The concerned reaction is $H_2 \rightarrow 2H$ ; $\Delta H = 436\ kJ\ mol^{-1}$

Energy required to bring 0.0409 moles of hydrogen gas to atomic state = 436 *0.0409 = **17.83 kJ**

Calculate of total number of hydrogen atom in 0.0409 mole of $H_2$ gas

1 mole of $H_2$ gas has $6.02 * 10^{23}$ molecules

0.0409 mole of $H_2$ gas = $6.02 * 10^{23}/1 * 0.0409$ molecules

Since 1 molecule of $H_2$ gas has 2 hydrogen atoms $6.02 * 10^{23} * 0.0409$ molecules of $H_2$ gas = $2 * 6.02 * 10^{23} * 0.0409 = 4.92 * 10^{22}$ atoms of hydrogen since energy required to excite an electron from the ground state to the next excited state is given by

$E = 13.6 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ eV = 13.6 * (1/1 – 1/4) = 13.6 * 3/4 = 10.2eV

= $1.632 * 10^{-21}$ kJ

Therefore energy required to excite $4.92 * 10^{22}$ electrons

= $1.632 * 10^{-21} * 4.92 * 10^{22}$ kJ = 8.03 * 10 = 80.3 kJ

Therefore total energy required = 17.83 + 80.3 = **98.17 kJ**

**SOL 18.**

For maximum energy, $n_1 = 1$ and $n_2 = \infty$

$1/\lambda = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Since $R_H$ is a constant and transition remains the same

$1/\lambda \propto Z^2$ ; $\lambda_{He}/\lambda_H = \frac{2_H^2}{2_{He}^2} = 1/4$

Hence, $\lambda_{He}$ = 1/4 * 91.2 = **22.8 nm**

**SOL 19.**

$\Psi_{2s}^2$ = probability of finding electron within 2s sphere $\Psi_{2s}^2 = 0$ (at node)

($\because$ Probability of finding an electron is zero at node)

$\therefore 0 = 1/32\pi\ (1/a_0)^3\ (2 - r_0/a_0)^2 .\ e^{-\frac{2r_v}{a_0}}$

(Squaring the given value of $\Psi_{2s}$)

or $[2 - r_0/a_0] = 0$; $\therefore 2 - r_0/a_0$ ; $\mathbf{2a_0} = r_0$

**SOL 20.**

$\lambda = h/mu = 6.627 * 10^{-34}/0.1 * 100$

or $\lambda$ = **$6.627 * 10^{-35}$ m = $6.627 * 10^{-25}$ Å**

**SOL 21.**

For hydrogen atom, Z = 1, n = 1

V = $2.18 * 10^6 * Z/n\ ms^{-1} = 2.18 * 10^6\ ms^{-1}$

de Broglie wavelength,

$\lambda = h/mv = 6.626 * 10^{-34} / 9.1 * 10^{-31} * 2.18 * 10^{6}$

$= \mathbf{3.34 * 10^{-10}\ m} = 3.3\ Å$

For 2p, $l = 1$

∴ Orbital angular momentum $= \sqrt{l(l + 1)}\ h/2\pi = \mathbf{\sqrt{2}\ h/2\pi}$