

# MATHEMATICS

## ONE MARKS QUESTIONS (1-20)

1. The dimension of the vector space  $V = \left\{ A = (a_{ij})_{n,n}; a_{ij} \in \mathbb{R}, a_{ij} = -a_{ji} \right\}$  over field  $\mathbb{R}$  is
  - a.  $n^2$
  - b.  $n^2 - 1$
  - c.  $n^2 - n$
  - d.  $\frac{n^2}{2}$
2. The minimal polynomial associated with the matrix  $\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$  is
  - a.  $x^3 - x^2 - 2x - 3$
  - b.  $x^3 - x^2 + 2x - 3$
  - c.  $x^3 - x^2 - 3x - 3$
  - d.  $x^3 - x^2 + 3x - 3$
3. For the function  $f(z) = \sin\left(\frac{1}{\cos(1/z)}\right)$ , the point  $z = 0$  is
  - a. a removable singularity
  - b. a pole
  - c. an essential singularity
  - d. a non-isolated singularity
4. Let  $f(z) = \sum_{n=0}^{15} z^n$  for  $z \in \mathbb{C}$ . If  $C: |z-i|=2$  then  $\int_C \frac{f(z) dz}{(z-i)^{15}} =$ 
  - a.  $2\pi i(1+15i)$
  - b.  $2\pi i(1-15i)$
  - c.  $4\pi i(1+15i)$
  - d.  $2\pi i$
5. For what values of  $\alpha$  and  $\beta$ , the quadrature formula  $\int_{-1}^1 f(x) dx \approx \alpha f(-1) + \beta f(1)$  is exact for all polynomials of degree  $\leq 1$ ?
  - a.  $\alpha = 1, \beta = 1$
  - b.  $\alpha = -1, \beta = 1$
  - c.  $\alpha = 1, \beta = -1$
  - d.  $\alpha = -1, \beta = -1$
6. Let  $f: [0, 4] \rightarrow \mathbb{R}$  be a three times continuously differentiable function. Then the value of  $f[1, 2, 3, 4]$  is
  - a.  $\frac{f''(\xi)}{3}$  for some  $\xi \in (0, 4)$
  - b.  $\frac{f''(\xi)}{6}$  for some  $\xi \in (0, 4)$
  - c.  $\frac{f'''(\xi)}{3}$  for some  $\xi \in (0, 4)$
  - d.  $\frac{f'''(\xi)}{6}$  for some  $\xi \in (0, 4)$
7. Which one of the following is TRUE?
  - a. Every linear programming problem has a feasible solution.
  - b. If a linear programming problem has an optimal solution then it is unique.
  - c. The union of two convex sets is necessarily convex.
  - d. Extreme points of the disk  $x^2 + y^2 \leq 1$  are the point on the circle  $x^2 + y^2 = 1$ .
8. The dual of the linear programming problem: Minimize  $c^T x$  subject to  $Ax \geq b$  and  $x \geq 0$  is
  - a. Maximize  $b^T w$  subject to  $A^T w \geq c$  and  $w \geq 0$
  - b. Maximize  $b^T w$  subject to  $A^T w \leq c$  and  $w \geq 0$
  - c. Maximize  $b^T w$  subject to  $A^T w \leq c$  and  $w$  is unrestricted
  - d. Maximize  $b^T w$  subject to  $A^T w \geq c$  and  $w$  is unrestricted
9. The resolvent kernel for the integral equation  $u(x) = F(x) + \int_{\log 2}^x f^{(x-t)} u(t) dt$  is
  - a.  $\cos(x-t)$
  - b. 1
  - c.  $e^{x-t}$

- d.  $e^{2i(\pi-t)}$
10. Consider the metrics  $d_1(f, g) = \left( \int_a^b |f(t) - g(t)|^2 dt \right)^{1/2}$  and  $d_\infty(f, g) = \sup_{t \in [a, b]} |f(t) - g(t)|$  on the space  $X = C[a, b]$  of all real valued continuous functions on  $[a, b]$ . Then which of the following is TRUE?
- Both  $(X, d_1)$  and  $(X, d_\infty)$  are complete.
  - $(X, d_1)$  is complete but  $(X, d_\infty)$  is NOT complete.
  - $(X, d_\infty)$  is complete but  $(X, d_1)$  is NOT complete.
  - Both  $(X, d_1)$  and  $(X, d_\infty)$  are NOT complete.
11. A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  need NOT be Lebesgue measurable if
- $f$  is monotone
  - $\{x \in \mathbb{R} : f(x) \geq \alpha\}$  is measurable for each  $\alpha \in \mathbb{R}$
  - $\{x \in \mathbb{R} : f(x) = \alpha\}$  is measurable for each  $\alpha \in \mathbb{R}$
  - For each open set  $G$  is  $\mathbb{R}$ ,  $f^{-1}(G)$  is measurable
12. Let  $\{e_n\}_{n=1}^\infty$  be an orthonormal sequence in a Hilbert space  $H$  and let  $x (\neq 0) \in H$ . Then
- $\lim_{n \rightarrow \infty} \langle x, e_n \rangle$  does not exist
  - $\lim_{n \rightarrow \infty} \langle x, e_n \rangle = \|x\|$
  - $\lim_{n \rightarrow \infty} \langle x, e_n \rangle = 1$
  - $\lim_{n \rightarrow \infty} \langle x, e_n \rangle = 0$
13. The subspace  $\mathbb{R} \times [0, 1]$  of  $\mathbb{R}^2$  (with the usual topology) is
- dense in  $\mathbb{R}^2$
  - connected
  - separable
  - compact
14.  $\mathbb{Z}[x] / \langle x^3 + x^2 + 1 \rangle$  is
- a field having 8 elements
  - a field having 9 elements
  - an infinite field
  - NOT a field
15. The number of element of a principal ideal domain can be
- 15
  - 25
  - 35
  - 36
16. Let,  $F, G$  and  $H$  be pair wise independent events such that  $P(F) = P(G) = P(H) = \frac{1}{3}$  and  $P(F \cap G \cap H) = \frac{1}{4}$ . Then the probability that at least one event among  $F, G$  and  $H$  occurs is
- $\frac{11}{12}$
  - $\frac{7}{12}$
  - $\frac{5}{12}$
  - $\frac{3}{4}$
17. Let  $X$  be a random variable such that  $E(X^2) = E(X) = 1$ . Then  $E(X^{100}) =$
- 0
  - 1
  - $2^{100}$
  - $2^{100} + 1$
18. For which of the following distributions, the weak law of large numbers does NOT hold?
- Normal
  - Gamma
  - Beta
  - Cauchy
19. If  $D = \frac{d}{dx}$  then the value of  $\frac{1}{(xD+1)}(x^{-1})$  is
- $\log x$
  - $\frac{\log x}{x}$
  - $\frac{\log x}{x^2}$
  - $\frac{\log x}{x^3}$
20. The equation

$$(\alpha xy^3 + y \cos x) dx + (x^2 y^2 + \beta) dy = 0$$

is exact for

- a.  $\alpha = \frac{3}{2}, \beta = 1$   
 b.  $\alpha = 1, \beta = \frac{3}{2}$   
 c.  $\alpha = \frac{2}{3}, \beta = 1$   
 d.  $\alpha = 1, \beta = \frac{2}{3}$

### TWO MARKS QUESTIONS (21-50)

21. If  $A = \begin{pmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{pmatrix}$ , then the

trace of  $A^{102}$  is

- a. 0  
 b. 1  
 c. 2  
 d. 3
22. Which of the following matrices is NOT diagonalizable?

- a.  $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$   
 b.  $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$   
 c.  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$   
 d.  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

23. Let  $V$  be the column space of the

matrix  $A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -1 \end{pmatrix}$ . Then the orthogonal

projection of  $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$  on  $V$  is

- a.  $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$   
 b.  $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$   
 c.  $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$   
 d.  $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

24. Let  $\sum_{n=-\infty}^{\infty} a_n (z-1)^n$  be the Laurent series expansion of  $f(z) = \sin\left(\frac{z}{z+1}\right)$ . Then

- $a_{-2} =$   
 a. 1  
 b. 0  
 c.  $\cos(1)$   
 d.  $-\frac{1}{2}\sin(1)$

25. Let  $u(x, y)$  be the real part of an entire function  $f(z) = u(x, y) + iv(x, y)$  for  $z = x + iy \in \mathbb{C}$ . If  $C$  is the positively oriented boundary of a rectangular region  $R$  in  $\mathbb{C}^2$ , then  $\oint_C \left[ \frac{\partial u}{\partial y} dx - \frac{\partial u}{\partial x} dy \right] =$

- a. 1  
 b. 0  
 c.  $2\pi$   
 d.  $\pi$

26. Let  $\phi: [0, 1] \rightarrow \mathbb{R}$  be three times continuously differentiable. Suppose that the iterates defined by  $x_{n+1} = \phi(x_n), n \geq 0$  converge to the fixed point  $\xi$  of  $\phi$ . If the order of convergence is three then

- a.  $\phi'(\xi) = 0, \phi''(\xi) = 0$   
 b.  $\phi'(\xi) \neq 0, \phi''(\xi) = 0$   
 c.  $\phi'(\xi) = 0, \phi''(\xi) \neq 0$   
 d.  $\phi'(\xi) \neq 0, \phi''(\xi) \neq 0$

27. Let  $f: [0, 2] \rightarrow \mathbb{R}$  be a twice continuously differentiable function. If  $\int_0^2 f(x) dx \approx 2f(1)$ , then the error in the approximation is

- a.  $\frac{f'(\xi)}{12}$  for some  $\xi \in (0, 2)$   
 b.  $\frac{f'(\xi)}{2}$  for some  $\xi \in (0, 2)$   
 c.  $\frac{f''(\xi)}{3}$  for some  $\xi \in (0, 2)$   
 d.  $\frac{f''(\xi)}{6}$  for some  $\xi \in (0, 2)$

28. For a fixed  $t \in \mathbb{R}$ , consider the linear programming problem:

$$\text{Maximize } z = 3x + 4y$$

$$\text{Subject to } x + y \leq 100$$

$$x + 3y \leq t$$

$$\text{and } x \geq 0, y \geq 0$$

The maximum value of  $z$  is 400 for  $t =$

- a. 50  
 b. 100  
 c. 200  
 d. 300
29. The minimum value of  $z = 2x_1 - x_2 + x_3 - 5x_4 + 22x_5$  subject to

$$x_1 - 2x_4 + x_5 = 6$$

$$x_2 + x_4 - 4x_5 = 3$$

$$x_3 + 3x_4 + 2x_5 = 10$$

$$x_j \geq 0, j = 1, 2, \dots, 5$$

is

- a. 28  
 b. 19  
 c. 10  
 d. 9
30. Using the Hungarian method, the optimal value of the assignment problem whose cost matrix is given by

5	23	14	8
10	25	1	23
35	16	15	12
16	23	11	7

is

- a. 29

- b. 52  
 c. 26  
 d. 44

31. Which of the following sequence  $\{f_n\}_{n=1}^{\infty}$  of functions does NOT converge uniformly on  $[0, 1]$ ?

a.  $f_n(x) = \frac{e^{-x}}{n}$

b.  $f_n(x) = (1-x)^n$

c.  $f_n(x) = \frac{x^2 + nx}{n}$

d.  $f_n(x) = \frac{\sin(nx+n)}{n}$

32. Let  $E = \{(x, y) \in \mathbb{R}^2 : 0 < x < y\}$ . Then

$$\iint_E y e^{-(x+y)} dx dy =$$

a.  $\frac{1}{4}$

b.  $\frac{3}{2}$

c.  $\frac{4}{3}$

d.  $\frac{3}{4}$

33. Let  $f_n(x) = \frac{1}{n} \sum_{k=0}^n \sqrt{k(n-k)} \binom{n}{k} x^k (1-x)^{n-k}$  for  $x \in [0, 1], n = 1, 2, \dots$ . If  $\lim_{n \rightarrow \infty} f_n(x) = f(x)$  for  $x \in [0, 1]$ , then the maximum value of  $f(x)$  on  $[0, 1]$  is

a. 1

b.  $\frac{1}{2}$

c.  $\frac{1}{3}$

d.  $\frac{1}{4}$

34. Let  $f: (C_{00}, \|\cdot\|_1, \|\cdot\|_1) \rightarrow \mathbb{R}$  be a non zero continuous linear functional. The number of Hahn-Banach extensions of  $f$  to  $(\ell^1, \|\cdot\|_1)$  is

- a. One  
 b. Two  
 c. Three

35. If  $I: (\mathbb{R}^2, \|\cdot\|_2) \rightarrow (\mathbb{R}^2, \|\cdot\|_1)$  is the identity map, then
- Both  $I$  and  $I^{-1}$  are continuous
  - $I$  is continuous but  $I^{-1}$  is NOT continuous
  - $I^{-1}$  is continuous but  $I$  is NOT continuous
  - Neither  $I$  and  $I^{-1}$  is continuous
36. Consider the topology  $\tau = \{G \subseteq \mathbb{R} : \mathbb{R} \setminus G \text{ is compact in } (\mathbb{R}, \tau_c)\} \cup \{\emptyset, \mathbb{R}\}$  on  $\mathbb{R}$ , where  $\tau_c$  is the usual topology on  $\mathbb{R}$  and  $\emptyset$  is the empty set. Then  $(\mathbb{R}, \tau)$  is
- a connected Hausdorff space
  - connected but NOT Hausdorff
  - hausdorff but NOT connected
  - neither connected nor Hausdorff
37. Let
- $$\tau_1 = \{G \subseteq \mathbb{R} : G \text{ is finite or } \mathbb{R} \setminus G \text{ is finite}\}$$
- and
- $$\tau_2 = \{G \subseteq \mathbb{R} : G \text{ is countable or } \mathbb{R} \setminus G \text{ is countable}\}$$
- Then
- neither  $\tau_1$  nor  $\tau_2$  is a topology on  $\mathbb{R}$
  - $\tau_1$  is a topology on  $\mathbb{R}$  but  $\tau_2$  is NOT a topology on  $\mathbb{R}$
  - $\tau_2$  is a topology on  $\mathbb{R}$  but  $\tau_1$  is NOT a topology on  $\mathbb{R}$
  - both  $\tau_1$  and  $\tau_2$  are topologies on  $\mathbb{R}$
38. Which one of the following ideals of the ring  $\mathbb{Z}[i]$  of Gaussian integers is NOT maximal?
- $\langle 1+i \rangle$
  - $\langle 1-i \rangle$
  - $\langle 2+i \rangle$
  - $\langle 3+i \rangle$
39. If  $Z(G)$  denotes the centre of a group  $G$ , then the order of the quotient group  $G/Z(G)$  cannot be
- 4
  - 6
  - 15
  - 25
40. Let  $\text{Aut}(G)$  denote the group of automorphism of a group  $G$ . Which one of the following is NOT a cyclic group?
- $\text{Aut}(\mathbb{Z}_4)$
  - $\text{Aut}(\mathbb{Z}_6)$
  - $\text{Aut}(\mathbb{Z}_8)$
  - $\text{Aut}(\mathbb{Z}_{10})$
41. Let  $X$  be a non-negative integer valued random variable with  $E(X^2) = 3$  and  $E(X) = 1$ . Then  $\sum_{i=1}^{\infty} iP(X \geq i) =$
- 1
  - 2
  - 3
  - 4
42. Let  $X$  be a random variable with probability density function  $f \in \{f_0, f_1\}$ , where
- $$f_0(x) = \begin{cases} 2x, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases} \text{ and}$$
- $$f_1(x) = \begin{cases} 3x^2, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$
- For testing the null hypothesis  $H_0: f = f_0$  against the alternative hypothesis  $H_1: f = f_1$  at level of significance  $\alpha = 0.19$ , the power of the most powerful test is
- 0.729
  - 0.271
  - 0.615
  - 0.385
43. Let  $X$  and  $Y$  be independent and identically distributed  $U(0, 1)$  random variables. Then  $P\left(Y < \left(X - \frac{1}{2}\right)^2\right) =$
- $\frac{1}{12}$
  - $\frac{1}{4}$
  - $\frac{1}{3}$
  - $\frac{2}{3}$

44. Let  $X$  and  $Y$  be Banach spaces and let  $T: X \rightarrow Y$  be a linear map. Consider the statements:

P: If  $x_n \rightarrow x$  in  $X$  then  $Tx_n \rightarrow Tx$  in  $Y$ .

Q: If  $x_n \rightarrow x$  in  $X$  and  $Tx_n \rightarrow y$  in  $Y$  then  $Tx = y$ .

Then

- P implies Q and Q implies P
  - P implies Q but Q does not imply P
  - Q implies P but P does not imply Q
  - Neither P implies Q nor Q implies P
45. If  $y(x) = x$  is a solution of the differential equation

$$y'' - \left( \frac{2}{x^2} + \frac{1}{x} \right) (xy' - y) = 0, 0 < x < \infty, \text{ then}$$

its general solution is

- $(\alpha + \beta e^{-2/x})x$
  - $(\alpha + \beta e^{2/x})x$
  - $\alpha x + \beta e^x$
  - $(\alpha e^x + \beta)x$
46. Let  $P_n(x)$  be the Legendra polynomial of degree  $n$  such that  $P_n(1) = 1, n = 1, 2, \dots$ . If

$$\int_{-1}^1 \left( \sum_{j=1}^n \sqrt{j(2j+1)} P_j(x) \right)^2 dx = 20, \text{ then } n =$$

- 2
  - 3
  - 4
  - 5
47. The integral surface satisfying the equation

$$y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = x^2 + y^2 \text{ and passing through}$$

the curve  $x = 1 - t, y = 1 + t, z = 1 + t^2$  is

- $z = xy + \frac{1}{2}(x^2 - y^2)^2$
  - $z = xy + \frac{1}{4}(x^2 - y^2)^2$
  - $z = xy + \frac{1}{8}(x^2 - y^2)^2$
  - $z = xy + \frac{1}{16}(x^2 - y^2)^2$
48. For the diffusion problem

$$u_{xx} = u_t (0 < x < \pi, t > 0), \quad u(0, t) = 0,$$

$u(\pi, t) = 0$  and  $u(x, 0) = 3 \sin 2x$ , the solution is given by

- $3e^{-t} \sin 2x$
  - $3e^{-4t} \sin 2x$
  - $3e^{-9t} \sin 2x$
  - $3e^{-2t} \sin 2x$
49. A simple pendulum, consisting of a bob of mass  $m$  connected with a string of length  $a$ , is oscillating in a vertical plane. If the string is making an angle  $\theta$  with the vertical, then the expression for the Lagrangian is given as

$$a. \quad ma^2 \left( \theta^2 - \frac{2g}{a} \sin^2 \left( \frac{\theta}{2} \right) \right)$$

$$b. \quad 2mga \sin^2 \left( \frac{\theta}{2} \right)$$

$$c. \quad ma^2 \left( \frac{\theta^2}{2} - \frac{2g}{a} \sin^2 \left( \frac{\theta}{2} \right) \right)$$

$$d. \quad \frac{ma}{2} \left( \theta^2 - \frac{2g}{a} \cos \theta \right)$$

50. The extremal of the functional

$$\int_0^1 \left( y + x^2 + \frac{y'^2}{4} \right) dx, y(0) = 0, y(1) = 0 \text{ is}$$

$$a. \quad 4(x^2 - x)$$

$$b. \quad 3(x^2 - x)$$

$$c. \quad 2(x^2 - x)$$

$$d. \quad x^2 - x$$

#### Common Data for Questions (51 & 52)

Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear transformation defined by

$$T(x_1, x_2, x_3) = (x_1 + 3x_2 + 2x_3, 3x_1 + 4x_2 + x_3, 2x_1 + x_2 - x_3)$$

51. The dimension of the range space of  $T^2$  is

- 0
- 1
- 2
- 3

52. The dimension of the null space of  $T^3$  is

- 0
- 1
- 2
- 3

**Common Data for Questions (53 & 54)**

Let  $y_1(x) = 1+x$  and  $y_2(x) = e^x$  be two solutions of  $y''(x) + P(x)y'(x) + Q(x)y(x) = 0$ .

53.  $P(x) =$
- $1+x$
  - $-1-x$
  - $\frac{1+x}{x}$
  - $\frac{-1-x}{x}$
54. The set of initial conditions for which the above differential equation has NO solution is
- $y(0) = 2, y'(0) = 1$
  - $y(1) = 0, y'(1) = 1$
  - $y(1) = 1, y'(1) = 0$
  - $y(2) = 1, y'(2) = 2$

**Common Data for Questions (55 & 56)**

Let  $X$  and  $Y$  be random variables having the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} e^{-\frac{x-y}{2y}}, & \text{if } -\infty < x < \infty, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

55. The variance of the random variable  $X$  is
- $\frac{1}{12}$
  - $\frac{1}{4}$
  - $\frac{7}{12}$
  - $\frac{5}{12}$
56. The covariance between the random variables  $X$  and  $Y$  is
- $\frac{1}{3}$
  - $\frac{1}{4}$
  - $\frac{1}{6}$
  - $\frac{1}{12}$

**Statement for Linked Answer Question (57 and 58)**

Consider the function  $f(z) = \frac{e^z}{z(z^2+1)}$ .

57. The residue of  $f$  at the isolated singular point in the upper half plane  $\{z = x+iy \in \mathbb{C} : y > 0\}$  is
- $-\frac{1}{2e}$
  - $-\frac{1}{e}$
  - $\frac{e}{2}$
  - $2$
58. The Cauchy Principal Value of the integral  $\int_{-\infty}^{\infty} \frac{\sin x dx}{x(x^2+1)}$  is
- $-2\pi(1+2e^{-1})$
  - $\pi(1+e^{-1})$
  - $2\pi(1+e)$
  - $-\pi(1+e^{-1})$

**Statement for Linked Answer Question (59 and 60)**

Let  $f(x, y) = kxy - x^2y - xy^2$  for  $(x, y) \in \mathbb{R}^2$ , where  $k$  is a real constant. The directional derivative of  $f$  at the point  $(1, 2)$  in the direction of the unit vector  $u = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$  is  $\frac{15}{\sqrt{2}}$ .

59. The value of  $k$  is
- $2$
  - $4$
  - $1$
  - $-2$
60. The value of  $f$  at a local minimum in the rectangular region  $R = \left\{ (x, y) \in \mathbb{R}^2 : |x| < \frac{3}{2}, |y| < \frac{3}{2} \right\}$  is
- $-2$
  - $-3$
  - $-\frac{7}{8}$
  - $0$