MATHEMATICS

1. The number of elements in a finite field is always a
   a. prime number
   b. even number
   c. number = p^n where $n \geq 1$ and p is a prime number
   d. multiple of 6

2. The value of $i^{1/3}$ are
   a. $-i, \frac{i \pm \sqrt{3}}{2}$
   b. $i, \frac{i \pm \sqrt{3}}{2}$
   c. $-i, \frac{\sqrt{3}i \pm 1}{2}$
   d. $i, \frac{\sqrt{3}i \pm 1}{2}$

3. If A and B are both sets having n elements then the number of surjective (onto) functions from A to B is
   a. n^2
   b. n^n
   c. n!
   d. n^2-n

4. If $A \xrightarrow{f} B \xrightarrow{g} C$ are functions such that the composite function gof : A→C is injective (one-one), then
   a. both f and g are injective
   b. g is injective but f need not be
   c. f is injective but g need not be
   d. neither f nor g need be injective

5. The least number of elements in a group having a proper non-abelian subgroup is
   a. 8
   b. 12
   c. 24
   d. 120

6. Let F be a field containing 11 elements. Which one of the following is correct?
   a. If α is a non-zero element of F, then $5\alpha \neq 0$
   b. $\alpha^5 = 1$ for every non-zero $\alpha \in F$ where 1 is the multiplicative identity of F
   c. $\alpha^{10} = 1$ for all $\alpha \in F$
   d. Let α be a non-zero element of F. It is possible to find a proper subset S of F such that $\alpha \in S$ and $\beta s \in S$ for any $\beta \in F$, $s \in S$

7. The number of subgroups of a cyclic group with 100 elements is
   a. 2
   b. 5
   c. [illegible]
   d. 10

8. Which one of the following is correct?
   a. Let $G = S_n$ be the permutation group on n symbols. Then for all σ, τ ∈ G $(\sigma\tau)^2 = \sigma^2\tau^2$
   b. If in a group G, $(xy)^2 = x^2y^2$ for all x, $y \in G$ then G must be cyclic
   c. Let $G = S_3$. Let $A = \{\sigma \in G : \sigma^3 = \text{identity}\}$. Then A is a subgroup of S_3.
   d. Let $G = S_4$. Let $B = \{\sigma \in S_4 : \sigma^4 = \text{identity but } \sigma^2 \neq \text{identity}\}$. Then B contains exactly 7 elements

9. The number of roots of the equation $x^6+x^5+x^4+x^3+x^2+x^1+1 = 0$ in Z_7 is
   a. 1
   b. 2
   c. 3
   d. 4

10. Which one of the following statements is correct where R[x] denotes the polynomial ring in the one variable x over a ring R:
    a. if R is a field then R[x] is a field
    b. if R is an integral domain, then R[x] is a field
    c. if R is a field then R[x] is an integral domain
    d. Every integral domain is a field

11. Which of the following rings are integral domains?

1. Z_{60}
2. Z_{71}
3. Z_{82}
4. Z_{97}

Select the correct answer using the codes given below:

a. 1 and 2
b. 2 and 3
c. 2 and 4
d. 3 and 4

12. The number of real solutions of the equation $x^2-3|x|+2=0$ is

a. zero
b. two
c. three
d. four

13. If α is a complex number such that $\alpha^2+\alpha+1=0$, then α^{31} is equal to

a. α
b. α^2
c. 1
d. 0

14. If $x = \frac{1+i}{\sqrt{2}}$, then the value of [illegible]$+x^4$[illegible]^2[illegible]1 will be

a. 0
b. 1
c. -1
d. i

15. If the ro[illegible]ts o[illegible] equation $x^3+px^2+qx+r=0$ are [illegible], then

a. [illegible]
b. $p^3 = q^3 r^2$
c. $p^3 = q^3 r$
d. $q^3 = p^3 r$

16. The equation whose roots are cubes of roots of the equation $x^3+x+1=0$ is

a. $x^3+3x^2+4x-1=0$
b. $x^3+3x^2+4x+1=0$
c. $x^3+3x^2-4x+1=0$
d. $x^3-3x^2+4x+1=0$

17. Let $S=\{p+q\sqrt{2} : p, q \in Q\}$. If '+' and '.' Are the usual operations of addition and multiplication of real numbers then (S, + , .) is

a. not a commutative ring
b. a commutative ring but not an integral domain
c. an integral domain but not a field
d. a field

18. If $Z = x + iy$ and $\left|\frac{1-iZ}{Z-i}\right| = 1$ [illegible] Z [illegible]s on

a. x- axis
b. y – axis
c. a circle with radius [illegible]ty
d. on lin[illegible]

19. Squar[illegible] of order n over R has ran[illegible] n. [illegible] one of the following state[illegible]ents [illegible] NOT correct?

a. [illegible] has rank n
b. [illegible] has n linearly independent columns
c. A is non-singular
d. A is singular

20. If C is a non-singular matrix and $B = C\begin{bmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{bmatrix} C^{-1}$ then

a. $B^2 = 1$
b. $B^2 = 0$
c. $B^3 = 1$
d. $B^3 = 0$

21. If $E(\theta) = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ θ and φ differ by an odd multiple of π/2, then $E(\theta)E(\varphi)$ is a

a. null matrix
b. unit matrix
c. diagonal matrix
d. none of these

22. Suppose $X = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : b - c = 4 \right\}$

$Y = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \middle| a = b + c \right\}$ and

$Z = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \middle| b = 0, c = d \right\}$

Which of these subsets of the vector space $\mathbb{R}^4$ is/are subspace(s)?

a. X only

b. Y and Z

c. X, Y and Z

d. X and Z

23. Let ω ($\neq$ 1) be a cube root of 1 in $\mathbb{C}$. Then the determinant of the matrix $\begin{bmatrix} 1 & 1 & \omega \\ 1 & 1 & \omega^2 \\ \omega^2 & \omega & 1 \end{bmatrix}$ is

a. ω

b. –3

c. $-\omega$

d. ω^2

24. Suppose P is an n × n matrix such that k^{th} diagonal element of PP^T is zero. Consider the following statements:

1. the k^{th} row of P is zero
2. the k^{th} row of PP^T is zero
3. the k^{th} column of P is zero
4. the k^{th} column of PP^T is zero

Select the correct answer using the codes given below:

a. 1 and 3

b. 1, 2 and 4

c. 2, 3 and 4

d. 1, 2 and 3

25. If $X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ then the rank of X^TX, where X^T denotes the transpose of X, is

a. 0

b. 2

c. 3

d. 4

26. Let T be a linear transformation from a 3-dimensional vector space V into a 2-dimensinal vector space W. Then T

a. can be both injective nor surjective

b. can be neither injective nor surjective

c. can be surjective but cannot be surjective

d. can be injective but cannot be surjective

27. Let W_1, W_2, W_3 be subspaces of a vectors space V such that $W_3 \subset W_1$. Then which one of the following is correct?

a. $W_1 \cap (W_2 + W_3) = W_2 + W_1 \cap W_3$

b. $W_1 \cap (W_2 + W_3) = W_3 + W_1 \cap W_2$

c. $W_1 \cup (W_2 + W_3) = W_3 + W_1 \cup W_3$

d. None of the above

28. Let ξ_1, ξ_2 and ξ_3 be vectors of a vector space V over the field F. If r and s are arbitrary elements of F and the set (ξ_1, ξ_2, $r\xi_1 + s\xi_2 + \xi_3$) is linearly dependent, then (ξ_1, ξ_2, ξ_3) is

a. linearly dependent set

b. a null set

c. linearly independent set

d. none of the above

29. Which of the following mapping $R^2 \to R^2$ is not a linear mapping?

a. $(x_1, x_2) \to (x_2, x_1)$

b. $(x_1, x_2) \to (x_1 + x_2, x_2)$

c. $(x_1, x_2) \to (x_1 + 1, x_2)$

d. $(x_1, x_2) \to (0, 0)$

30. R is the set of real numbers. In R^2, let f_1 and f_2 the two transformation defined by

$f_1(x, y) = (0, y)$

$f_2(x, y) = (y, x)$

Then the product of the mapping $f_2 \circ f_1(x, y)$ gives the projection of x-y plane on the

a. y- axis

b. x-axis

c. Line y = x

d. Line y= -x

31. The dimension of the subspace of $\mathbb{R}^3$ spanned by (-3, 0, 1), (1, 2, 1) and (3, 0, -1)

a. 0

b. 1

c. 2

d. 3

32. If V is a vector space over an infinite field F such that dim V= 2, then the number of distinct subspaces V has is

a. 2

b. 3

c. 4

d. infinite

33. Consider the following linear transformation from the vector space R^2 into the vector space R^3:

T(x, y)=(-x, -y, 3x +8y, 9x – 11y)

Then the rank and nullity of T are respectively

a. 2 and 0

b. 1 and 3

c. 2 and 1

d. 2 and 3

34. $\vec{a}$, $\vec{b}$, $\vec{c}$ are vectors of magnitude 3,4 and 5 respectively. If $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ to $\vec{c}+\vec{a}$ and $\vec{c}$ to $\vec{a}+\vec{b}$, then the magnitude of $(\vec{a}+\vec{b}+\vec{c})$ is

a. $-5\sqrt{2}$

b. $5\sqrt{2}$

c. 7

d. 0

35. If the two vectors are collinear ,then their vector product is

a. One

b. the product of their moduli

c. equal to zero

d. none of the above

36. $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})$ is equal to

a. $3\vec{a}$

b. $5\vec{a}$

c. $-\vec{a}$

d. $2\vec{a}$

37. The plane 2x- 2y + z + 12 =0 touches the sphere $x^2+y^2+z^2-2x-4y+2z = 3$, then the point of contact is

a. (1, 4, 1)

b. -1, 4, -2)

c. (1, 4, 2)

d. (-1, -4, -2)

38. The vertical angle of the cone $x^2+y^2-z^2=0$ is

a. $\pi/4$

b. $\pi/3$

c. $\pi/2$

d. $2\pi/3$

39. If t is real number, then the vector equation of straight line AB through two given points A and B with position vectors $\vec{\alpha}$ and $\vec{\beta}$, respectively is

a. $\vec{r}=(1-t)\vec{\alpha}+t\vec{\beta}$

b. $\vec{r}=\vec{\alpha}+t\vec{\beta}$

c. $\vec{r}=(1+t)\vec{\alpha}+t\vec{\beta}$

d. $\vec{r}=\vec{\alpha}-t\vec{\beta}$

40. If $\vec{a}$, $\vec{b}$, $\vec{a}+\vec{b}$ are the vectors of the same magnitude all vectors being proper vectors, then the angle between $\vec{a}$ and $\vec{b}$ is

a. 0

b. $\pi/2$

c. $\pi/3$

d. $2\pi/3$

41. The equation of the cylinder whose generators are parallel to the line x/1 = y/1 = z/1 and whose guiding curve is the ellipse $x^2+2y^2=1$, z= 0 is

a. $x^2+y^2+z^2=1$

b. $x^2+y^2+2z^2+yz+xz=0$

c. $x^2+2y^2+3z^2-4yz-2xz=1$

d. $x^2+y^2+z^2+yz+zx=0$

42. The points (1, -2), (3, 2), (1, 2) and (-1, 0) are the vertices of a

a. Rectangle

b. Square

c. Parallelogram

d. None of the above

43. If e and e′ are the eccentricities of two conjugate hyperbolas , then

a. $e^2+e'^2=1$

b. $e^2+e'^2=1$

c. $\frac{1}{e^2}+\frac{1}{e'^2}=1$

d. $\frac{1}{e^2}+\frac{1}{e'^2}=\frac{1}{2}$

44. The value of $|\vec{a}\times\vec{b}|$ is

a. $\sqrt{(\vec{a}.\vec{a})(\vec{b}.\vec{b})-(\vec{a}.\vec{b})^2}$

b. $\sqrt{(\vec{a}.\vec{a})(\vec{b}.\vec{b})-2(\vec{a}.\vec{b})^2}$

c. $\sqrt{\vec{a}^2+\vec{b}^2+2(\vec{a}.\vec{b})}$

d. $\sqrt{(\vec{a}.\vec{a})(\vec{b}.\vec{b})+(\vec{a}.\vec{b})^2}$

45. The line $3x + 4y - 24 = 0$ cuts the x-axis at A and y-axis at B. Then the in center of the triangle OAB, where O is the origin is

a. (1, 2)

b. (2, 2)

c. (12, 12)

d. (2, 12)

46. The equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle is

a. $x^2+y^2+z^2-2x+4y-6z+10=0$

b. $x^2+y^2+z^2-2x-4y+4z+7=0$

c. $x^2+y^2+z^2-6x-4y-4z+9=0$

d. $x^2+y^2+z^2-6x+4y+4z-9=0$

47. $|\vec{a}\times\vec{b}| = |\vec{a}| + |\vec{b}|$, where $\vec{a}$ and $\vec{b}$ are any vectors. This equality

a. always holds

b. never holds

c. holds only where $\vec{a}=\vec{b}=0$

d. holds only when $\vec{a}=k\vec{b}$, $k\geq 0$ or one of $\vec{a}$ and $\vec{b}$ is zero

48. If $\vec{a}$ is the position vector of a given point A and $\vec{r}$ is the position vector of any point P in space with respect to origin O then the equation $(\vec{r}-\vec{a}).\vec{r}=0$ representing the locus of the point P is

a. the sphere with OA as diameter

b. the plane which passes through A and is perpendicular to OA

c. the straight line OA

d. the plane perpendicular to the axis of α

49. For the given sequence $\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$ which one of the following statements is correct?

a. Limit superior = limit inferior

b. Neither limit superior nor limit inferior exists

c. Limit superior = 1 and limit inferior = -1

d. Limit superior = 1 and limit inferior = 0

50. Asymptote of the curve $x^3+y^3-3axy=0$ is

a. $x+y+a=0$

b. $x+y-a=0$

c. $x+a=0$

d. $y+a=0$

51. Consider the following statements:

1. the length of the subnormal to the parabola $y^2=4ax$ at any point is 2a
2. the length of the subtangent to the parabola $y^2=4ax$ varies as the abscissa of the point of contact.
3. The equation of the tangent at the origin to the parabola $y^2=4ax$ is x-axis.

Which of the statements are correct?

a. 1, 2 and 3

b. 1 and 2

c. 1 and 3

d. 2 and 3

52. Let $f(x)=x^2-4x+3$. The following statements are associated with f:

1. f is increasing in $(2,\infty)$
2. f is decreasing in $(-\infty,-2)$
3. f has a stationary point at $x=2$

Which of these statements are correct?

a. 1 and 2

b. 1 and 3

c. 2 and 3

d. 1, 2 and 3

53. The value of c in Rolle's theorem, where $-\pi/2 < c < \pi/2$ and $f(x)=\cos x$, is equal to

a. $\pi/4$

b. $\pi/3$

c. π

d. 0

54. Let f(x) = $x^p \sin 1/x$, when x ≠ 0 and f(x) = 0 , when x = 0 Then f(x) can be differentiated at the origin provided p is

a. Any real number
b. Zero
c. One
d. An integer greater than 1

55. A function f(x) is differentiable in the closed and bounded interval [p, q] . The differential f'(p) and f'(q) are such that f'(p) < 0 and f'(q) > 0. Then f(x) will assume on [p, q]

a. neither infimum nor supremum
b. supremum and not infimum
c. infimum and not supremum
d. none of the above

56. Consider the following statements:

1. a differentiable function is continuous
2. a continuous function is differentiable
3. a continuous function on a closed interval [a, b] of finite length is uniformly continuous on [a,b]

Which of these statements are correct?

a. 1, 2 and 3
b. 2 and 3
c. 1 and 3
d. 1 and 2

57. Let f(x) be defined on [0, 5] by

$$f(x)=\begin{cases} x, \text{ if } x \text{ is a rational number} \\ 5-x, \text{ if } x \text{ is an irrational number} \end{cases}$$

Then the function is continuous in the interval at

a. No point
b. All the points
c. 2 points
d. one point

58. A ⊂ R, R is the field of real numbers. F(A) denote the collection of all real valued functions defined on A. The two functions g, f ∈ F(A) are such that they satisfy the algebraic operation (f + g) x= f(x)+ g(x), x ∈ A. Then with respect to this operation the set F(A) is

a. Always closed
b. Never closed
c. Closed only for selected functions
d. None of the above

59. Given the following limits:

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

2. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = 1$

3. $\lim_{x \to 0} \frac{\log(x+1)}{x} = 1$

Which of these are correct?

a. 1, 2 and 3
b. 1 and 2
c. 1 and 3
d. 2 and 3

60. For a bounded function f(x) the upper and lower bounds are M and m and |f(x)| is R-integrable on the interval [a, b]. Given $\int_a^b f(x)dx = \mu(b-a)$ such that m≤μ≤M, μ in [a,b] can

a. always be determined
b. never be determined
c. only be conditionally determined
d. be determined only for a < μ < b

61. For the curve

x = f'(θ) sin θ + f''(θ) cos θ

and y = f'(θ) cos θ- f''(θ) sin θ,

Where derivatives are with respect to θ, the length of arc is

a. (θ) + f''(θ) + C
b. f'(θ)+f '(θ)+C
c. (θ)-f''(θ)+C
d. f'(θ)-f''(θ)+C

62. The length of the curve y =log sec x from x = 0 to x = π/3 is equal to

a. log (3 + $\sqrt{2}$)
b. log (2+ $\sqrt{3}$)
c. log (x + $\sqrt{3}$)
d. log (2 cos x)

63. For the function f(x, y) = $x^5F(y/x)$,the value of the differential $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is equal to

a. $5x^5F(y/x)$

b. $5x^4F(y/x)$

c. $4x^5F(y/x)$

d. $4x^4F(y/x)$

64. The only double point of the curve $(x-2)^2 - y(y-1)^2 = 0$ is :

a. $\left(2, \frac{1}{3}\right)$

b. (0, 0)

c. (2, 1)

d. (2, 2)

65. If $u = x^2 - y^2 + z^2$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to:

a. 4u

b. u^2

c. u

d. 2u

66. Let $A_n = \int_0^{\pi/4} \tan^n x \, dx$. Then the value of $A_{10} + A_8$ is

a. 1/8

b. 1/9

c. 1/10

d. −1/9

67. The arc of parabola $y^2 = 4ax$ [illegible]twee[illegible] th[illegible] points, where the latus rectum [illegible]nd a [illegible]ine parallel to the latus rectum and a[illegible] twice the distance of latus rect[illegible]m [illegible]om [illegible]e vertex meet the parabola, is r[illegible]lved about the x-axis. Then the area o[illegible] [illegible]e surface of revolution so for[illegible]ed is

a. $\pi\sqrt{3}$

b. $2\pi/3$[illegible]$\sqrt{3}$ - [illegible]

c. [illegible]$\sqrt{3}$ - $2\sqrt{2}$)

d. [illegible]$(3\sqrt{3} - 2\sqrt{2})$

68. [illegible]en the sequence $\left(\frac{n}{n+1}\right)_{n=1}^{\infty}$ and an arbitrarily small positive number ε. Then the value of a positive integer N such that $\left|\frac{n}{n+1} - 1\right| < \varepsilon$ whenever $n \geq N$ must satisfy the relation

a. $N \leq \left(\frac{1}{\varepsilon} - 1\right)$

b. $N < \left(\frac{1}{\varepsilon} - 1\right)$

c. $N > \left(\frac{1}{\varepsilon} - 1\right)$

d. $N \geq \left(\frac{1}{\varepsilon} - 1\right)$

69. The general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \frac{e^{2x}}{x}$ is

a. $C_1 + C_2 x \log x - \frac{x^2}{2}e^{2x}$

b. $C_1 + C_2 x + x \log x$ [illegible]

c. $C_1 e^{2x} + C_2 e^{-2x} +$ [illegible] $\log - \frac{x^2}{2}$

d. $C_1 e^{2x}$ [illegible] $\log x - x$

70. The [illegible]on of the equation $\frac{d^2y}{dx^2}$ [illegible] $\frac{dy}{dx}$ [illegible] $24y = 0$ with $y(0) = 0$, $y'(0) = 3$ is

a. [illegible] $+ e^{-2x}$

b. $e^x - e^{-2x}$

c. $e^x - e^{2x}$

d. $e^x + e^{2x}$

71. The singular solution of the differential equation $y = xp + a\sqrt{1+p^2}$ is a

a. parabola

b. hyperbola

c. circle

d. straight line

72. The general solution of the differential equation $(\sin y - y \sin xy)\,dx + (x \cos y - x \sin xy)\,dy = 0$ is

a. $x \cos y + \sin xy = \lambda$

b. $x \cos y - \sin xy = \lambda$

c. $x \sin y + \cos xy = \lambda$

d. $x \sin y - \cos xy = \lambda$

73. The solution of the equation $\frac{dy}{dx} = \frac{2x - y + 1}{x + 2y - 3}$ is

a. $xy + y^2 + x^2 + 3y - x = c$

b. $xy + y^2 - x^2 - 3y - x = c$

c. $xy + y^2 + x^2 - 3y - x = c$

d. $xy + y^2 - x^2 - 3y + x = c$

74. If in a culture of yeast ,y_0 is the amount of y east present initially, and the rate of growth dy/dt is proportional to the amount y at time t and if y doubles in one day then the amount expected after 3 days equals

a. $3y_0$

b. $8y_0$

c. $9y_0$

d. $27y_0$

75. If the rate of growth is proportional to the amount x of the substance present and dx/dt = kx, then x is equal to (with C_1 constant)

a. C_1e^{-kt}

b. C_1e^{kt}

c. C_1e^{-2kt}

d. C_1e^{2kt}

76. The general solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\cos x$ is

a. $(C_1+C_2x + \sin x)e^{-x}$

b. $(C_1+C_2x - \sin x)e^{-x}$

c. $(C_1+C_2x + \cos x)e^{-x}$

d. $(C_1+C_2x - \cos x)e^{-x}$

77. The orthogonal trajectories of the family $y^2= 4ax + 4a^2$ is the family

a. $x^2 = 4ay + 4a^2$

b. $y^2=4ay + 4a^2x$

c. $y^2= 4ax + 4a^2$

d. $x^2=4ax + 4a^2$

78. A singular solution of the differential equation $y^2[1+(dy/dx)^2]= R^2$ is

a. $y = R/2$

b. $y = R$

c. $y = 3R/2$

d. $y = 2R$

79. Two scale pans each of mass 4kg are connected by string passing over a pulley. Two masses whose sum is 10 kg are to be placed in the two scale pans so as to produce an acceleration of 8/9(g= acceleration due to gravity). Two masses are

a. 1 kg and 9 kg

b. 2 kg and 8kg

c. 3kg and 7kg

d. 4 kg and 6kg

80. Two equal rods each of length 2a and weight W are freely; jointed at one end. The system is placed in a vertical position with the other ends of the two rods on a smooth horizontal plane and the middle points of the rods joined by a string of length l which is kept taut in equilibrium position. The tension of the string is

a. $\frac{Wl}{2a}$

b. $\frac{Wa}{\sqrt{4a^2 - l^2}}$

c. $\frac{2Wa}{\sqrt{4a^2 - l^2}}$

d. $\frac{Wl}{\sqrt{4a^2 - l^2}}$

81. The velocity of a boat relative to water $2\vec{i}$ + […] and that of water relative to earth is $\vec{i}$ […] 2 […], where $\vec{i}$, $\vec{j}$ represent velocities of one km on hour due East and North respectively. The velocity of boat relative to earth us

a. 3 km an hour due East

b. $2\sqrt{2} + \sqrt{5}$ km on hour at 45^0 North of East

c. $\sqrt{17}$ km an hour at $\tan^{-1}$ (4) North of East

d. $\sqrt{17}$ km an hour at $\tan^{-1}$(4) due North

82. A ladder 5 meter long rests on a rough horizontal plane and against a smooth vertical wall. The ladder is about to slip on the rough plane when its lower end is at a distance 4 metres from the wall. The coefficient of friction is

a. 2/3

b. 1/2

c. 3/5

d. 4/5

83. C.G. of any three equal particles placed at the vertices of a triangle is at the

a. incentre of the triangle

b. orthocentre of the triangle

c. circumcentre of the triangle

d. centroid of the triangle

84. A particle acted upon by three forces whose magnitudes are proportional to(i) 3,5,8;(ii) 5, 7, 10; and (iii) 5, 7, 14; can be arranged at rest

a. only in case (ii)

b. only in case (i)

c. only in cases (i) and (ii)

d. in all the three cases

85. Two parallel like forces P and Q act at a distance α apart and have a resultant which acts at a point C. A couple of moment M and arm of length β is combined with these like parallel forces. The resultant will be force

a. $P+Q+\frac{M}{\alpha}$ acting at C

b. P+Q acting at a distance M/(P+Q) from C

c. P+Q acting at a distance $\frac{\alpha M}{\beta(P+Q)}$ from C

d. P+Q acting at a distance $\frac{M}{\alpha}$ from C

86. P and Q are two like parallel forces. If P is moved parallel to itself through a distance of 4 cm, then the resultant of P and Q moves through a distance of

a. 2 cm

b. 4 cm

c. $\frac{4Q}{P+Q}$ cm

d. $\frac{4P}{P+Q}$ cm

87. The resultant of the forces P and Q is R. If Q is doubled then R gets doubled in magnitude ,R is again doubled if Q is reversed. Then P^2, Q^2 and R^2 are in the ratio

a. 2 : [illegible] : 3

b. 3 : 2 : 2

c. 2 : 3 : 2

d. 2 : 3 : 3

88. Two forces P and Q where P= 2Q acts at a point in directions at right angles to each other. Their resultant will be inclined to P at angle θ such that

a. $0 < \theta < \pi/6$

b. $\pi/6 < \theta < \pi/4$

c. $\pi/4 < \theta < \pi/3$

d. $\pi/3 < \theta < \pi/2$

89. A stone falling vertically from rest travels half of its total path in the last second of its fall. The total time of its fall is

a. less than 2 seconds

b. equal to 2 seconds

c. greater than 2 seconds but less than 3 seconds

d. greater than 3 seconds

90. An elastic ball hits a horizontal floor vertically with a speed of u and is allowed to strike the floor another two times after which it rebounds from the floor with a velocity 27/64 u. Neglecting the resistance of air, the coefficient of elasticity between the ball the floor is

a. [illegible]5

b. 3/4

c. [illegible]3

d. [illegible]/16

91. A particle of unit mass oscillates in a straight path under a force of attraction proportional to its distance from a fixed point O. If its velocity at distance a, b from O be v_1 and v_2 respectively, then the period of oscillation is

a. $2\pi\sqrt{\frac{b^2-a^2}{v_1^2-v_2^2}}$

b. $2\pi\sqrt{\frac{v_1^2-v_2^2}{b^2-a^2}}$

c. $2\pi\sqrt{\frac{b^2-a^2}{v_1^2-v_2^2}}$

d. $2\pi\sqrt{\frac{b^2}{v_2^2}+\frac{a^2}{v_1^2}}$

92. Consider the algorithm :

1. i ← 10
2. i ← (i – 1)
3. if i > 0, go to step 2
4. Print i

The looping of statements 2-3 during execution, occurs

a. 9 times

b. 10 times

c. 11 times

d. 12 times

93. 51 letters of a certain language are to be uniquely coded using strings of 6 bits. In this system, how many possible non-printable control characters can be uniquely coded?

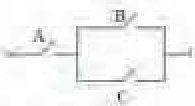
a. 0

b. 8

c. 13

d. 77

94.



Which of the following logical operations is equivalent to the given connection?

a. A+(B . C)

b. B.C

c. A.(B+C)

d. B+C

95. Given the bit strings p = 11011 and q =10101, which of the following is the value of (NOT p) AND q?

a. 00100

b. 11111

c. 10101

d. 10001

96. Given that the binary equivalent of decimal 12 is stored in one-byte register. Which one of the following is the content of this register after the shift-left operation is performed 3 times?

a. 0000[illegible]101

b. 0010[illegible]000

c. [illegible]00

d. [illegible]00000

97. Assertion(A): If the position of the resultant of two parallel forces which are like, remains unaltered when the forces are interchanged, the forces are reciprocal of each other.

Reason (R): The resultant of two like parallel forces P and Q acting at A and B respectively acts at a point C on AB such that P × AC = Q × BC.

a. Both A and B are true and R is the correct explanation of A

b. Both A and R are true but R is not a correct explanation of A.

c. A is true but R is false

d. A is false but R is true

98. Let n ≥ 3 and let the complex numbers $\alpha_1, \alpha_2, \ldots, \alpha_n$ be the roots of $x^n - 1$ with $\alpha_1 =$ 1.

Assertion (A): For any positive integer t>1, $(t-\alpha_2)(t-\alpha_3)\ldots(t-\alpha_n)$ is again a positive integer.

Reason (R): For any positive integer t, $(t-\alpha_2)(t-\alpha_3)\ldots(t-\alpha_n)$ equals $(t^{n-1}-1)$.

a. Both A and B are true and R is the correct explanation of A

b. Both A and R are true but R is not a correct explanation of A.

c. A is true but R is false

d. A is false but R is true

99. Let q_1, q_2 be non-zero rational numbers. Let $q_1 = n_1/m$ and $q_2 = n_2/m$ where $n_1, n_2 \in Z$, $m \in Z$ and m>0. Let H = $\{xq_1 + yq_2 : x, y \in Z\}$

(Z denotes the set of all integers)

Assertion (A): H forms a subgroup of additive group of rationals and is cyclic.

Reason (R): Any subgroup of a cyclic group is cyclic H = Z_q where q = l/m and l = least common multiple of n_1 and n_2.

a. Both A and B are true and R is the correct explanation of A

b. Both A and R are true but R is not a correct explanation of A.

c. A is true but R is false

d. A is false but R is true

100. Assertion (A): $(189)_{10} = (10111101)_2$.

Reason (R): The binary number can be expressed in powers of its base 2 as $\sum_{i=1}^{8} b_i 2^{i-1}$ where b_i is the I^{th} bit (from the right).

a. Both A and B are true and R is the correct explanation of A

b. Both A and R are true but R is not a correct explanation of A.

c. A is true but R is false

d. A is false but R is true