Remainder Problems
with HCF and LCM

Recap

- Method:
  - Prime Factorization
  - Divisibility tests
- Problem Keywords
  - LCM: Minimum number, Least amount, Smallest duration etc.
  - HCF: Maximum number, Most amount, Longest duration etc.

HCF Remainder Problems

4 Types

Greatest Number Which Divides *x*, *y* and *z*?

HCF Type 1 (Simple)

- Lets start with a simple example: 12, 18, 30

These slides accompany the YouTube Video Tutorial: https://www.youtube.com/watch?v=CMLB4QN801Q

Greatest Number Which Divides *x*, *y* and *z* Leaves Same Remainder *r* (Given)?

HCF Type 2 (Same Remainder- Given)

- Lets find a number which divides all 14, 20 and 32 leaving remainder 2

Greatest Number Which Divides *x*, *y* and *z* Leaves Same Remainder ~~*r*~~ (Not Given)?

HCF Type 3 (Same Remainder- Not Given)

- Number which divides all 14, 20 and 32 leaving same remainder

Greatest Number Which Divides *x*, *y* and *z*, Leaving Remainders *a*, *b* and *c* (Respectively) HCF Type 4 (Different Remainder- Given)

- Number which divides 12, 18 and 30 leaving remainder 2, 3 and 0.

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HCF Problems – 4 Types Summary Understand and Remember

1. Greatest number which divides x, y and z = HCF (x, y, z)
2. Greatest number which divides x, y and z and leaves remainder r = HCF$(x - r, y - r, z - r)$
3. Greatest number which divides x, y and z and leaves same remainder = HCF$(|x - y|, |y - z|, |z - x|)$
4. Greatest number which divides x, y and z and leaves remainder a, b, c = HCF$(x - a, y - b, z - c)$

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LCM Remainder Problems

4 Types

Smallest Number Divisible by *x*, *y* and *z*?

LCM Type 1 (Simple)

- Lets start with a simple example: 6, 9, 12

Smallest/Largest Number of *n* Digits Divisible by *x*, *y*, *z*? LCM Type 2 (Multiples of LCM)

- Smallest/Largest number of 3 digits divisible by 6, 9, 12

Smallest Number When Divided by *x*, *y* and *z* Leaves Same Remainder *r* (Given)? LCM Type 3 (Same Remainder)

- Number divisible by 6, 9, 12 leaves remainder 2

Smallest Number When Divided by *x*, *y* and *z* Leaves Remainder *a*, *b*, *c*?

LCM Type 4 (Different Remainder)

- ***x - a = y - b = z - c = common difference d***

- Smallest number divided by 2, 3, 4, 5, 6 leaves remainder 1, 2, 3, 4, 5

LCM Problems – 4 Types Summary Understand and Remember

1. Smallest number divisible by *x*, *y* and *z* = LCM(*x*, *y*, *z*)
2. Smallest number of *n* digits divisible by *x*, *y* and *z* = Multiple of LCM(*x*, *y*, *z*)
3. Smallest number when divided by *x*, *y* and *z* leaves same remainder *r* = LCM(*x*, *y*, *z*) + *r*
4. Smallest number when divided by *x*, *y* and *z* leaves remainder *a*, *b*, *c*
   - *x - a = y - b = z - c* = common difference *d*
   - LCM *(a, b, c) - d*

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Variations of LCM (Understand)

- Smallest/Largest number of *n* digits when divided by *x*, *y* and *z* leaves same remainder *r* = Multiple of LCM(*x*, *y*, *z*) + *r*

- Smallest/Largest number of *n* digits when divided by *x*, *y* and *z* leaves remainder *a*, *b*, *c* = Multiple of LCM(*x*, *y*, *z*) – d (*x-a = y-b = z-c = common difference d)*

Example - 1

- Find the greatest number of 5-digits which on being divided by 9, 12, 24 and 45 leaves 3, 6, 18 and 39 as remainders respectively.

Example - 2

- Find the smallest number which, on being added 23 to it, is exactly divisible by 32, 36, 48 and 96.

Example – 3 (Advanced)

- When dividing a number by 12, 15 or 48 there will always be a remainder of 10. If the number is the least possible, how many divisors does the number have?

- Number of divisors of $p^a q^b$ (p, q primes) is (a + 1) (b + 1): *Application of Combination*

Generalization – Chinese Remainder Theorem Next Class!!

- Find the smallest number which when divided by 7, 9, 11 produces 1, 2, 3 as reminders
    - 7 – 1 = 6
    - 9 – 2 = 7
    - 11 – 3 = 8
  - But
    - 7 – 2 × 1 = 5
    - 9 – 2 × 2 = 5
    - 11 – 2 × 3 = 5