A FILL IN THE BLANKS

1. $y = 10^x$ is the reflection of $y = \log_{10} x$ in the line whose equation is... **(IIT 1982; 2M)**

2. The set of lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$ is concurrent at the point... **(IIT 1982; 2M)**

3. If a, b and c are in A.P., then the straight line $ax + by + c = 0$ will always pass through a fixed point whose coordinates are (.......) **(IIT 1984; 2M)**

4. The sides AB, BC and CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices is... **(IIT 1984;2M)**

5. The set of all real numbers a such that $a^2 + 2a$, $2a + 3$ and $a^2 + 3a + 8$ are the sides of a triangle is... **(IIT 1985; 2M)**

6. The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in quadrant number... **(IIT 1985; 2M)**

7. Let the algebraic sum of the perpendicular distance from the points $(2, 0)$, $(0, 2)$ and $(1, 1)$ to a variable straight line be zero; then the line passes through a fixed point whose coordinates are... **(IIT 1991;2M)**

8. The vertices of a triangle are $A(-1, -7)$, $B(5, 1)$ and $C(1, 4)$. The equation of the bisector of the angle $\angle ABC$ is... **(IIT 1993; 2M)**

B TRUE / FALSE

1. The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$. **(IIT 1983; 1M)**

2. No tangent can be drawn from the point $(5/2, 1)$ to the circumcircle of the triangle with vertices $(1, \sqrt{3})$, $(1, -\sqrt{3})$, $(3, \sqrt{3})$. **(IIT 1985; 1M)**

3. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$, then the two triangles with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) and (a_1, b_1), (a_2, b_2), (a_3, b_3) must be congruent. **(IIT 1985; 1M)**

4. The lines $2x + 3y + 19 = 0$ and $9x + 6y - 17 = 0$ cut the coordinate axes in concyclic points. **(IIT 1988; 1M)**

C OBJECTIVE QUESTIONS

➡ ***Only one option is correct :***

1. Given the four lines with the equations

$$x - 2y - 3 = 0, 3x + 4y - 7 = 0,$$
$$2x + 3y - 4 = 0, 4x + 5y - 6 = 0$$

then : **(IIT 1980)**

(a) they are all concurrent

(b) they are the sides of a quadrilateral

(c) none of these

2. The point $(4, 1)$ undergoes the following three transformations successively :

(i) Reflection about the line $y = x$

(ii) Transformation through a distance 2 units along the positive direction of x-axis.

(iii) Rotation through an angle $\frac{\pi}{4}$ about the origin in the counter clockwise direction.

Then the final position of the point is given by the coordinates : **(IIT 1980; 1M)**

(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $(-\sqrt{2}, 7\sqrt{2})$

(c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $(\sqrt{2}, 7\sqrt{2})$

3. The straight lines $x + y = 0$, $3x + y - 4 = 0$, $x + 3y - 4 = 0$ form a triangle which is : **(IIT 1983; 1M)**

(a) isosceles (b) equilateral

(c) right angled (d) none of these

4. The points $\left(0, \frac{8}{3}\right)$, $(1, 3)$ and $(82, 30)$ are vertices of : **(IIT 1986; 2M)**

(a) an obtuse angled triangle
(b) an acute angled triangle
(c) a right angled triangle
(d) none of these

5. If $P = (1, 0)$, $Q = (-1, 0)$ and $R = (2, 0)$ are three given points, then locus of the points satisfying the relation $SQ^2 + SR^2 = 2SP^2$, is : **(IIT 1988; 2M)**

(a) a straight line parallel to x-axis
(b) a circle passing through the origin
(c) a circle with the centre at the origin
(d) a straight line parallel to y-axis

6. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then : **(IIT 1990; 2M)**

(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

(c) $a^2 + p^2 = b^2 + q^2$ (d) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

7. If the sum of the distance of a point from two perpendicular lines in a plane is 1, then its locus is : **(IIT 1992; 2M)**

(a) square
(b) circle
(c) straight line
(d) two intersecting lines

8. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then $PQRS$ must be a : **(IIT 1998; 2M)**

(a) rectangle (b) square
(c) cyclic quadrilateral (d) rhombus

9. If $P(1, 2)$, $Q(4, 6)$, $R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then : **(IIT 1998; 2M)**

(a) $a = 2, b = 4$ (b) $a = 3, b = 4$
(c) $a = 2, b = 3$ (d) $a = 3, b = 5$

10. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/(are) always rational point(s) : **(IIT 1998; 2M)**

(a) centroid (b) incentre
(c) circumcentre (d) orthocentre

(A rational point is a point both of whose co-ordinates are rational numbers)

11. Let $A_0, A_1, A_2, A_3, A_4, A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is : **(IIT 1998; 2M)**

(a) $\frac{3}{4}$ (b) $3\sqrt{3}$

(c) 3 (d) $\frac{3\sqrt{3}}{2}$

12. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is : **(IIT 2000)**

(a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$
(c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$

13. The incentre of the triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is : **(IIT 2000)**

(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$

(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

14. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is : **(IIT 2001)**

(a) 2 (b) 0
(c) 4 (d) 1

15. Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals : **(IIT 2001)**

(a) $\frac{|m + n|}{(m - n)^2}$ (b) $\frac{2}{|m + n|}$

(c) $\frac{1}{|m + n|}$ (d) $\frac{1}{|m - n|}$

16. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. Then the equations of the bisector of the angle PQR is : **(IIT 2002)**

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$

(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$

17. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. Then the point O divides the segment PQ in the ratio : **(IIT 2002)**

(a) 1 : 2 (b) 3 : 4
(c) 2 : 1 (d) 4 : 3

18. Let $O(0, 0)$, $P(3, 4)$, $Q(6, 0)$ be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are : **(IIT 2007)**

(a) $\left(\frac{4}{3}, 3\right)$ (b) $\left(3, \frac{2}{3}\right)$

(c) $\left(3, \frac{4}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{2}{3}\right)$

D OBJECTIVE QUESTIONS

➡ *More than one options are correct :*

1. Three lines $px+qy+r=0$, $qx+ry+p=0$ and $rx+py+q=0$ are concurrent, if : (IIT 1985; 2M)
   (a) $p+q+r=0$ (b) $p^2+q^2+r^2=pr+rq$
   (c) $p^3+q^3+r^3=3pqr$ (d) none of these

2. All points lying inside the triangle formed by the points (1, 3), (5, 0) and (– 1, 2) satisfy : (IIT 1986; 2M)
   (a) $3x+2y\ge 0$ (b) $2x+y-13\ge 0$
   (c) $2x-3y-12\le 0$ (d) $-2x+y\ge 0$
   (e) none of these

3. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x+y=1$. If the intercepts made by the circle $x^2+y^2-x+3y=0$ on L_1 and L_2 are equal, then which of the following equation can represent L_1? (IIT 1999; 3M)
   (a) $x+y=0$ (b) $x-y=0$
   (c) $x+7y=0$ (d) $x-7y=0$

E SUBJECTIVE QUESTIONS

1. A straight line L is perpendicular to the line $5x-y=1$. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L. (IIT 1980)

2. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line $y=2x+c$. Find c and the remaining vertices. (IIT 1981; 4M)

3. The ends A, B of a straight line segment of constant length c slide upon the fixed rectangular axes OX, OY respectively. If the rectangle $OAPB$ be completed, then show that the locus of the foot of the perpendicular drawn from P to AB is
$$x^{2/3}+y^{2/3}=c^{2/3}$$
(IIT 1983; 2M)

4. The coordinates of A, B, C are (6, 3), (–3, 5), (4, –2) respectively and P is any point (x, y). Show that the ratio of the areas of the triangles ΔPBC and ΔABC is
$$\left|\frac{x+y-2}{7}\right|.$$
(IIT 1983; 2M)

5. The vertices of a triangle are
$$[at_1t_2, a(t_1+t_2)], [at_2t_3, a(t_2+t_3)], [at_3t_1, a(t_3+t_1)].$$
Find the orthocentre of the triangle. (IIT 1983; 3M)

6. Two equal sides of an isosceles triangle are given by the equations $7x-y+3=0$ and $x+y-3=0$ and its third side passes through the point (1, –10). Determine the equation of the third side. (IIT 1984; 4M)

7. Two sides of a rhombus $ABCD$ are parallel to the lines $y=x+2$ and $y=7x+3$. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on the y-axis, find possible coordinates of A. (IIT 1985; 5M)

8. One of the diameter of the circle circumscribing rectangle $ABCD$ $4y=x+7$. If A and B are the points (–3, 4) and (5, 4) respectively, then find the area of rectangle. (IIT 1985; 3M)

9. The equations of the perpendicular bisectors of the sides AB and AC of a triangle ABC are $x-y+5=0$ and $x+2y=0$, respectively. If the point A is (1, –2), find the equation of the line BC. (IIT 1986; 5M)

10. Lines $L_1\equiv ax+by+c=0$ and $L_2\equiv lx+my+n=0$ intersect at the point P and make an angle θ with each other. Find the equation of a line L different from L_2 which passes through P and makes the same angle θ with L_1. (IIT 1988; 5M)

11. Let ABC be a triangle with $AB=AC$. If D is mid-point of BC, the foot of the perpendicular drawn from D to AC and F the mid-point of DE. Prove that AF is perpendicular to BE. (IIT 1989; 5M)

12. A line cuts the x-axis at A (7, 0) and the y-axis at B (0, – 5). A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R find the locus of R. (IIT 1990; 4M)

13. Straight lines $3x+4y=5$ and $4x-3y=15$ intersect at the point A. Points B and C are chosen on these two lines such that $AB=AC$. Determine the possible equations of the line BC passing through the point (1, 2). (IIT 1990; 4M)

14. Find the equation of the line passing through the point (2, 3) and making intercept of length 3 units between the lines $y+2x=2$ and $y+2x=5$. (IIT 1991; 4M)

15. Determine all values of α for which the point (α, α^2) lies inside the triangles formed by the lines $2x+3y-1=0$, $x+2y-3=0$, $5x-6y-1=0$ (IIT 1992; 6M)

16. A line through $A(-5,-4)$ meets the line $x+3y+2=0$, $2x+y+4=0$ and $x-y-5=0$ at the points B, C and D respectively. If $(15/AB)^2+(10/AC)^2=(6/AD)^2$, find the equation of the line. (IIT 1993; 5M)

17. A rectangle $PQRS$ has its side PQ parallel to the line $y=mx$ and vertices PQ and S on the lines $y=a$, $x=b$ and $x=-b$, respectively. Find the locus of the vertex R. (IIT 1996; 2M)

18. A variable straight line of slope 4 intersects the hyperbola $xy=1$ at two points. Find the locus of the point which divides the line segment between these two point in the ratio 1 : 2. (IIT 1997; 5M)

19. Using co-ordinate geometry, prove that the three altitudes of any triangle are concurrent. (IIT 1998)

20. For points $P=(x_1, y_1)$ and $Q=(x_2, y_2)$ of the co-ordinate plane, a new distance $d(P, Q)$ is defined by $d(P,Q)=|x_1-x_2|+|y_1-y_2|$.

Let $O=(0,0)$ and $A=(3,2)$. Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. **(IIT 2000; 10M)**

21. A straight line L through the origin meets the line $x+y=1$ and $x+y=3$ at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to $2x-y=5$ and $3x+y=5$ respectively. Lines L_1 and L_2 intersect at R, show that the locus of R as L varies, is a straight line. **(IIT 2002; 5M)**

G ASSERTION AND REASON

This question contains STATEMENT-I (Assertion) and STATEMENT-II (Reason).

1. Lines $L_1: y-x=0$ and $L_2: 2x+y=0$ intersect the line $L_3: y-2=0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **(IIT 2007)**

Statement-I : The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

Because

Statement-II : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(b) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(c) Statement-I is True, Statement-II is False
(d) Statement-I is False, Statement-II is True

ANSWERS

A Fill in the Blanks

1. $y=x$ 2. $\left(\frac{3}{4},\frac{1}{2}\right)$ 3. $(1,-2)$ 4. 205 5. $a>5$ 6. $\left(\frac{41}{7},\frac{22}{7}\right)$ 7. $(1,1)$ 8. $7y-x+2$

B True / False

1. True 2. True 3. False 4. True

C Objective Questions (Only one option)

1. (c) 2. (c) 3. (a) 4. (d) 5. (d) 6. (b) 7. (a)
8. (d) 9. (c) 10. (a) 11. (c) 12. (d) 13. (d) 14. (a)
15. (d) 16. (c) 17. (b) 18. (c)

D Objective Questions (More than one option)

1. (a, c) 2. (a, c) 3. (b, c)

E Subjective Questions

1. $x+5y=\pm 5\sqrt{2}$ 2. $c=-4, (4,4), (4,0)$ 5. $(-a, a(t_1+t_2+t_3+t_1t_2t_3))$

6. $x-3y-31=0$ and $3x+y+7=0$ 7. $\left(0,\frac{5}{2}\right), (0,0)$ 8. 32 sq. units

9. $14x+23y-40=0$ 10. $2(al+bm)(ax+by+c)-(a^2+b^2)(lx+my+n)=0$ 12. $x^2+y^2-7x+5y=0$

13. $x-7y+13=0$ and $7x+y+9=0$ 14. $x=2$ and $3x+4y=18$ 15. $-\frac{3}{2}<\alpha<-1 \cup \frac{1}{2}<\alpha<1$

16. $2x+3y+22=0$ 17. $(m^2-1)x-my+b(m^2+1)+am=0$ 18. $(2y-x)(2x-y)=-4$

G Assertion and Reason

1. (b)

SOLUTIONS

A FILL IN THE BLANKS

1. $y=10^x$ is reflection of $y=\log_{10}x$ about $y=x$

2. The set of lines $ax+by+c=0$, where $3a+2b+4c=0$ or $\frac{3}{4}a+\frac{1}{2}b+c=0$ are concurrent at $\left(x=\frac{3}{4}, y=\frac{1}{2}\right)$ i.e. comparing the coefficients of x and y

Thus, point of concurrency is $\left(\frac{3}{4},\frac{1}{2}\right)$

Alter As ; $ax+by+c=0$, satisfy $3a+2b+4c=0$ which represents system of concurrent lines whose point of concurrency could be obtained by comparison as,

$$ax + by + c = \frac{3a}{4} + \frac{2}{4}b + c$$

$\Rightarrow$ $x = 3/4$, $y = 1/2$ is point of concurrency

$\therefore$ $\left(\frac{3}{4}, \frac{1}{2}\right)$ is required point.

3. Since a, b, c are in A.P

$\therefore$ $2b = a + c$

or $a - 2b + c = 0$ which satisfy $ax + by + c = 0$

$\therefore$ $ax + by + c = 0$ always pass through a fixed point $(1, -2)$.

4. Since AB, BC, CA contains 3, 4 and 5 points

$\Rightarrow$ number of Δ's $= {}^{12}C_3 - {}^{3}C_3 - {}^{4}C_3 - {}^{5}C_3$

$= 220 - 1 - 4 - 10 = 220 - 15 = 205$

5. Since $a^2 + 2a, 2a + 3$ and $a^2 + 3a + 8$ forms sides of a Δ,

$\Rightarrow$ $a^2 + 3a + 8 < (a^2 + 2a) + (2a + 3)$

$\Rightarrow$ $a^2 + 3a + 8 < a^2 + 4a + 3$

or $a > 5$ .... (1)

also, $(a^2 + 3a + 8) + (2a + 3) > a^2 + 2a$

$\Rightarrow$ $3a > -11$

$a > -\frac{11}{3}$ ...(2)

and $(a^2 + 3a + 8) + (a^2 + 2a) > 2a + 3$

$\Rightarrow$ $2a^2 + 3a + 5 > 0$

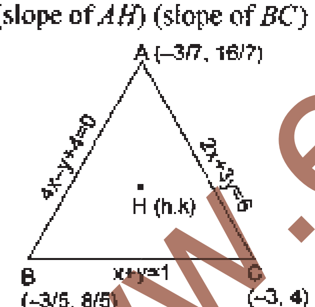
which is always true

$\therefore$ Δ is formed if,

$a > 5$

6. Let $H(h, k)$ be orthocentre

$\Rightarrow$ (slope of AH) (slope of BC) $= -1$



$\Rightarrow$ $\left(\frac{k - \frac{16}{7}}{h + \frac{3}{7}}\right) \cdot (-1) = -1$

$k - \frac{16}{7} = h + \frac{3}{7}$

$h - k = -\frac{19}{7}$ ... (1)

also, (slope of CH) (slope of AB) $= -1$

$\Rightarrow$ $\frac{k - 4}{h + 3} \cdot (4) = -1$

$\Rightarrow$ $4k - 16 = -h - 3$

$\Rightarrow$ $h + 4k = 13$ ...(2)

Solving equations (1) and (2), we get

$h = \frac{41}{7}, k = \frac{22}{7}$

$\therefore$ orthocentre $\left(\frac{41}{7}, \frac{22}{7}\right)$

7. Let the variable straight line be $ax + by + c = 0$ ... (1)

where algebraic sum of perpendiculars from (2, 0), (0, 2) and (1, 1) is zero

$\Rightarrow$ $\frac{2a + 0 + c}{\sqrt{a^2 + b^2}} + \frac{0 - 2b + c}{\sqrt{a^2 + b^2}} + \frac{a + b + c}{\sqrt{a^2 + b^2}} = 0$

$\Rightarrow$ $3a - 3b + 3c = 0$

or $a + b + c = 0$ ... (2)

$\therefore$ From (1) and (2) $ax + by + c = 0$ always passes through a fixed point (1, 1).

8. Equation of the line AB is $y - 1 = \frac{1 - (-7)}{5 - (-1)}(x - 5)$

$\Rightarrow$ $y - 1 = \frac{8}{6}(x - 5)$

$\Rightarrow$ $y - 1 = \frac{4}{3}(x - 5)$

$\Rightarrow$ $3y - 3 = 4x - 20$

$\Rightarrow$ $3y - 4x + 17 = 0$

Next, equation of the line BC is

$y - 4 = \frac{4 - 1}{1 - 5}(x - 1)$

$\Rightarrow$ $y - 4 = -\frac{3}{4}(x - 1)$

$\Rightarrow$ $4y - 16 = -3x + 3$

$\Rightarrow$ $3x + 4y - 19 = 0$

Again equation of the bisectors of the angles between two given lines AB and BC are

$\frac{3y - 4x - 17}{\sqrt{3^2 + 4^2}} = \pm \frac{4y + 3x - 19}{\sqrt{4^2 + 3^2}}$

$\Rightarrow$ $3y - 4x + 17 = \pm(4y + 3x - 19)$

$\Rightarrow$ $3y - 4x + 17 = 4y + 3x - 19$

and $3y - 4x + 17 = -(4y + 3x - 19)$

$\Rightarrow$ $36 = y + 7x$ and $7y - x = 2$

Of these two, the equation of the bisector of angle ABC is $7y = x + 2$

B TRUE / FALSE

1. (T) The point of intersection of $x + 2y = 10$ and $2x + y + 5 = 0$ is $\left(\frac{-20}{3}, \frac{25}{3}\right)$ which clearly satisfy $5x + 4y = 0$

2. (T) As, $(1, -\sqrt{3})$ and $(3, \sqrt{3})$ forms a right angle Δ.

$\therefore$ Equation of Circumcircle taking $(3, \sqrt{3})$ and $(1, -\sqrt{3})$ as end points of diameter.

$$(x-3)(x-1)+(y-\sqrt{3})(y+\sqrt{3})=0$$

$$\Rightarrow \quad x^2-4x+3+y^2-3=0$$

$$\Rightarrow \quad x^2+y^2-4x=0$$

clearly $(5/2,1) \Rightarrow \dfrac{25}{4}+1-10<0$

$\therefore$ $(5/2,1)$ lies inside the circle

Hence, no tangent can be drawn.

Thus, (True)

3. (F) As, $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$

represents Δ's are equal, which does not implies Δ's are congruent. Thus, (false)

4. (T) As, $a_1x+b_1y-c_1=0$ and $a_2x+b_2y-c_2=0$ cuts the coordinate axes at concyclic points.

$\Rightarrow \quad a_1a_2=b_1b_2$ or $a_1b_2+b_1a_2=0$

Here, $2x+3y+19=0$ and $9x+6y-17=0$

$\Rightarrow (a_1=2, b_1=3, c_1=19)$ and $(a_2=9, b_2=6, c_2=-17)$

$\therefore \quad a_1a_2=18$ and $b_1b_2=18$

$\Rightarrow a_1a_2=b_1b_2$. Thus pt's are concyclic

C Objective (Only One Option)

1. Here, $x+2y-3=0$ and $3x+4y-7=0$ intersect at $(1,1)$, which does not satisfy $2x+3y-4=0$ and $4x+5y-6=0$ also, $3x+4y-7=0$ and $2x+3y-4=0$ intersect at $(5,-2)$ which does not satisfy $x+2y-3=0$ and $4x+5y-6=0$

   Lastly, intersection point of $x+2y-3=0$ and $2x+3y-4=0$ is $(-1, 2)$ which satisfy $4x+5y-6=0$. Hence, only three lines are concurrent.

2. Let B, C, D be the position of the point $A(4,1)$ after the three operations I, II and III respectively. Then B is $(1,4)$, $C(1+2,4)$ *i.e.*, $(3,4)$. The point D is obtained from C by rotating the co-ordinate axes through an angle $\pi/4$ in anticlockwise direction.

   Therefore the co-ordinates of D are given by

$$X=3\cos\frac{\pi}{4}-4\sin\frac{\pi}{4}=-\frac{1}{\sqrt{2}}$$

$$Y=3\sin\frac{\pi}{4}+4\cos\frac{\pi}{4}=\frac{7}{\sqrt{2}}$$

   $\therefore$ Co-ordinates of D are $\left(-\dfrac{1}{\sqrt{2}}, \dfrac{7}{\sqrt{2}}\right)$

3. The point of intersection of three lines are $A(1,1)$, $B(2,-2)$, $C(-2,2)$.

$$\Rightarrow |AB|=\sqrt{1+9}=\sqrt{10}, |BC|=\sqrt{16+16}=4\sqrt{2},$$
$$|CA|=\sqrt{9+1}=\sqrt{10}$$

   $\therefore$ Δ is isosceles.

4. Since for $(0,8/3)$, $(1,3)$ and $(82,30)$

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} 0 & \frac{8}{3} & 1 \\ 1 & 3 & 1 \\ 82 & 30 & 1 \end{vmatrix}=0$$

   $\therefore$ Points are collinear.

   $\therefore$ (d) is correct answer.

5. Let, $S(x, y)$

$$\therefore \quad SQ^2+SR^2=2SP^2$$

$$\Rightarrow \quad (x+1)^2+y^2+(x-2)^2+y^2 = 2\{(x-1)^2+y^2\}$$

$$\Rightarrow \quad x^2+2x+1+y^2+x^2-4x+4+y^2 = 2\{x^2-2x+1+y^2\}$$

$$\Rightarrow \quad 2x+3=0$$

$$\Rightarrow \quad x=-\frac{3}{2}$$

   or a straight line parallel to y-axis.

6. Since, the origin remains the same. So, length of the perpendicular from the origin on the line in its position $\dfrac{x}{a}+\dfrac{y}{b}=1$ and $\dfrac{x}{p}+\dfrac{y}{q}=1$ are equal. Therefore,

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}$$

$$\Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$$

7. By the given conditions we can take two perpendicular lines as x and y-axis if (h,k) is any point on the locus, then $|h|+|k|=1$. Therefore the locus is $|x|+|y|=1$. This consist of a square of side 1. Hence the required locus is a square.

   Therefore, (a) is the answer.

8. Slope of $x+3y=4$ is $-1/3$

   and slope of $6x-2y=7$ is 3.

   Therefore, these two lines are perpendicular which show that both diagonals are perpendicular. Hence, $PQRS$ must be a rhombus.

9. $PQRS$ is a parallelogram if and only if the mid-point of PR is same as that of the mid-point of QS. That is, if and only if

$$\frac{1+5}{2}=\frac{4+a}{2} \text{ and } \frac{2+7}{2}=\frac{6+b}{2}$$

   $\Rightarrow a=2$ and $b=3$. Therefore, (c) is the answer.

10. Since the coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2-y_3}{3}\right)$ the centroid is always a rational point.
Therefore. (*a*) is the answer.

11. $$A_0 A_1^2 = \left(1-\frac{1}{2}\right)^2 + \left(0-\frac{\sqrt{3}}{2}\right)^2$$
$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4}+\frac{3}{4} = 1$$
$$\Rightarrow \quad A_0 A_1 = 1$$
Next, $$A_0 A_2^2 = \left(1+\frac{1}{2}\right)^2 + \left(0-\frac{\sqrt{3}}{2}\right)^2$$
$$= \left(\frac{3}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{9}{4}+\frac{3}{4} = \frac{12}{4} = 3$$
$$\Rightarrow \quad A_0 A_2 = \sqrt{3}$$
Next $$A_0 A_4^2 = \left(1+\frac{1}{2}\right)^2 + \left(0+\frac{\sqrt{3}}{2}\right)^2$$
$$= \left(\frac{3}{2}\right)^2 + \left(\frac{3}{4}\right) = \frac{9}{4}+\frac{3}{4} = \frac{12}{4} = 3$$
$$\Rightarrow \quad A_0 A_4 = \sqrt{3}$$
Thus, $(A_0 A_1)(A_0 A_2)(A_0 A_4) = 3.$

12. S is the mid-point of Q and R

therefore $S = \frac{7+6}{2}, \frac{3-1}{2} = 13/2, 1$

Now, slope of $PS = m = \frac{2-1}{2-13/2} = \frac{2}{9}$

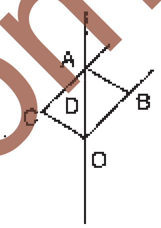
Now, equation of the line passing through $(1, -1)$ and parallel to PS is
$$y+1 = -\frac{2}{9}(x-1)$$
or $\quad 2x+9y+7=0$
Therefore, (*d*) is the answer.

13. Here, $AB = BC = CA = 2$
Therefore, it is an equilateral triangle.
⇒ The incentre cocides with centroid
Therefore, centroid
$$I = \left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right)$$
$$\Rightarrow \quad (1, 1/\sqrt{3})$$
Therefore, (*d*) is the answer.

14. $\left.\begin{array}{l} 3x+4y=9 \\ y=mx+1 \end{array}\right\}$ Solving for x,
$$x = \frac{5}{3+4m}$$
Now, for x to be an integer, $3+4m = \pm 5$ or ± 1
The integral values of m satisfying these conditions are -2 and -1. Therefore (*a*) is the answer.

15. Let $OB : y = mx$
$$CA : y = mx+1$$
$$BA : y = nx+1$$
$$OC : y = nx$$
The point of intersection B of OB and AB has x coordinate $\frac{1}{m-n}$

Now, area of a parallelogram
$OBAC = 2 \times$ area of $\Delta\, OBA$
$$= 2 \times \frac{1}{2} \times OA \times DB$$
$$= 2 \times \frac{1}{2} \times \frac{1}{m-n}$$
$$= \frac{1}{m-n} = \frac{1}{|m-n|}$$



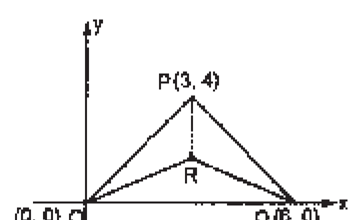
depending upon whether $m > n$ or $m < n$.
Therefore, (d) is the answer.

16. The line segment QR makes an angle of 60° with the positive direction of x-axis.
So, the bisector of the angle PQR will make an angle of 60° with the negative direction of x-axis it will therefore have angle of inclination of 120° and so its equation is,
$$y - 0 = \tan 120°(x - 0)$$
$$\Rightarrow \quad y = -\sqrt{3}x$$
$$\Rightarrow \quad y + \sqrt{3}x = 0$$

17. The ratio will be same if we take the perpendicular line segment.
Now, distance of origin from $4x+2y-9=0$ is
$$\frac{|-9|}{\sqrt{4^2+2^2}} = \frac{9}{\sqrt{20}}$$
and distance of origin from $2x+y+6=0$
is $$\frac{|6|}{\sqrt{2^2+1^2}} = \frac{6}{\sqrt{5}}$$

Hence, the required ratio $= \frac{9/\sqrt{20}}{6/\sqrt{5}} = \frac{3}{4}$

18. Since, Δ is isosceles, hence centroid is the desired point.



∴Coordinates of $R\left(3, \frac{4}{3}\right)$

D OBJECTIVE (MORE THAN ONE OPTION)

1. (a) (c) $px + qy + r = 0$

$$qx + ry + p = 0$$

and $rx - py + q = 0$ are concurrent

$$\Rightarrow \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$\Rightarrow (p+q+r)(p^2+q^2+r^2-pq-qr-pr) = 0$$

$$\Rightarrow p^3+q^3+r^3-3pqr = 0.$$

2. As, $3x + 2y \geq 0$ ...(1)

where (1, 3) (5, 0) and (−1, 2) satisfy (1)

again, $2x + y - 13 \geq 0$

is not satisfied by (1, 3), ∴(b) is false

$$2x - 3y - 12 \geq 0,$$

is satisfied for all points, ∴(c) is true

$$-2x + y \geq 0,$$

is not satisfied by (5, 0), ∴(d) is false. Thus (a), (c) are correct answers.

3. Let equation of line L_1 be $y = mx$. Intercepts made by L_1 and L_2 on the circle will be equal *i. e.* L_1 and L_2 are at the same distance from the centre of the circle.
Centre of the given circle is (1/2, −3/2). Therefore

$$\frac{|1/2 - 3/2 - 1|}{\sqrt{1+1}} = \left|\frac{\frac{m}{2} + \frac{3}{2}}{\sqrt{m^2+1}}\right|$$

$$\Rightarrow \frac{2}{\sqrt{2}} = \frac{|m+3|}{2\sqrt{m^2+1}}$$

$$8m^2 + 8 = m^2 + 6m + 9$$

$$7m^2 - 6m - 1 = 0$$

$$7m^2 - 7m + m - 1 = 0$$

$$\Rightarrow (7m+1)(m-1) = 0$$

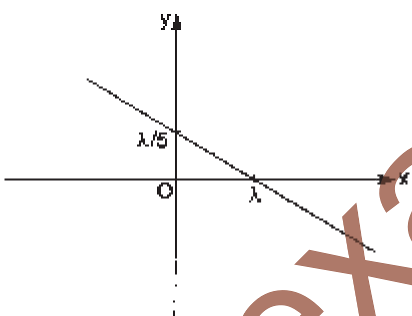
$$\Rightarrow m = -\frac{1}{7}$$

$$m = 1$$

Thus two chords are $x + 7y = 0$ and $y - x = 0$
Therefore, (b) and (c) are the answer

E SUBJECTIVE QUESTIONS

1. A straight line perpendicular to $5x - y = 1$ is $x + 5y = \lambda$, where, area of Δ = 5



$$\Rightarrow \frac{1}{2}\left|\lambda \cdot \frac{\lambda}{5}\right| = 5$$

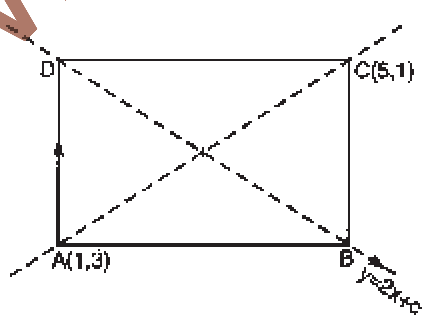
$$\Rightarrow \lambda^2 = 50$$

or $|\lambda| = 5\sqrt{2}$

∴ Equation of the line L is,

$$x + 5y = \pm 5\sqrt{2}.$$

2. As, diagonals of rectangle bisect each other, so mid-point of (1, 3) and (5, 1) must satisfy $y = 2x + c$ *i.e.*, (3, 2) lies on it.



$$\Rightarrow \quad 2 = 6 + c$$

or $c = -4$.

∴ Other two vertices lies on $y = 2x - 4$.

let $B(x, 2x - 4)$

∴ Slope of AB. Slope of $BC = -1$

$$\left(\frac{2x-4-3}{x-1}\right)\cdot\left(\frac{2x-4-1}{x-5}\right) = -1$$

$$\Rightarrow (x^2 - 6x + 8) = 0 \Rightarrow x = 4, 2.$$

∴ (4, 4), (4, 0) are required points.

3. Let $OA = a$ and $OB = b$. Then, the co-ordinates of A and B are $(a, 0)$ and $(0, b)$ respectively.

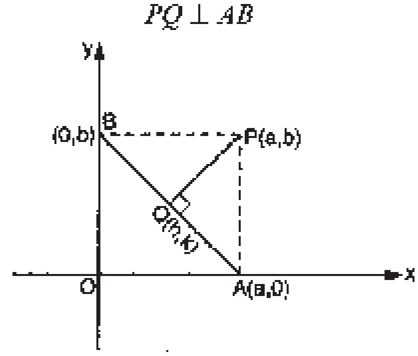
The co-ordinates of P are (a, b). Let θ be the foot of perpendicular from P on AB and let the co-ordinates of $Q(h, k)$. Here a and b are the variable and we have to find locus of Q.

Now, $AB = c$

$$\Rightarrow AB^2 = c^2 \Rightarrow OA^2 + OB^2 = c^2$$

$$\Rightarrow a^2 + b^2 = c^2 \quad ...(1)$$

Now, $PQ \perp AB$



⇒ slope of AB. slope of $PQ = -1$

$$\Rightarrow \quad \frac{k-b}{h-a}\cdot\frac{0-b}{a-0} = -1$$

$$\Rightarrow \quad bk - b^2 = ah - a^2$$

$$\Rightarrow \quad ah - bk = a^2 - b^2 \qquad \ldots(2)$$

Equation of line AB is $\frac{x}{a} + \frac{y}{b} = 1$

Since, Q lies on AB,

$$\frac{h}{a} + \frac{k}{b} = 1 \Rightarrow bh + ak = ab \qquad \ldots(3)$$

Solving (2) and (3), we get

$$\frac{h}{ab^2 + a(a^2 - b^2)} = \frac{k}{-b(a^2 - b^2) + a^2 b}$$

$$= \frac{1}{a^2 + b^2}$$

$$\Rightarrow \quad \frac{h}{a^3} = \frac{k}{b^3} = \frac{1}{c^2} \qquad \{\text{using (1)}\}$$

$$\Rightarrow \quad a = (hc^2)^{1/3} \quad \text{and} \quad b = (kc^2)^{1/3}$$

Substituting the values of a and b in $a^2 + b^2 = c^2$, we get $h^{2/3} + k^{2/3} = c^{2/3}$

or $\quad x^{2/3} + y^{2/3} = c^{2/3}$ required locus.

4. $$\frac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC} = \frac{\frac{1}{2}|x(5+2) + (-3)(-2-y) + 4(y-5)|}{\frac{1}{2}|6(5+2) + (-3)(-2-3) + 4(3-5)|}$$

$$= \frac{|7x + 7y - 14|}{|42 + 15 - 8|} = \frac{7|x + y - 2|}{49}$$

$$= \left|\frac{x + y - 2}{7}\right|$$

5. Let ABC be a Δ whose vertices are $A(at_1t_2, a(t_1 + t_2))$, $B(at_2t_3, a(t_2 + t_3))$ and $C(at_1t_3, a(t_1 + t_3))$.

Then,

$$\text{slope of } BC = \frac{a(t_2 + t_3) - a(t_1 + t_3)}{at_2t_3 - at_1t_3} = \frac{1}{t_3}$$

$$\text{Slope of } AC = \frac{a(t_1 + t_3) - a(t_1 - t_2)}{at_1t_3 - at_1t_2} = \frac{1}{t_1}$$

So, the equation of a line through A perpendicular to BC is,

$$y - a(t_1 + t_2) = -t_3(x - at_1t_2) \qquad \ldots(1)$$

and the equation of a line through B perpendicular to AC is,

$$y - a(t_2 + t_3) = -t_1(x - at_2t_3) \qquad \ldots(2)$$

The point of intersection of (1) and (2), is the orthocentre.

Subtracting (2) from (1), we get $x = -a$.

putting $x = -a$ in (1), we get

$$y = a(t_1 + t_2 + t_3 + t_1t_2t_3)$$

Hence, the co-ordinates of the orthocentre are $(-a, a(t_1 + t_2 + t_3 + t_1t_2t_3))$.

6. The equation of any line passing through $(1, -10)$ is $y + 10 = m(x - 1)$. Since it makes equal angles, say θ, with the given lines, therefore

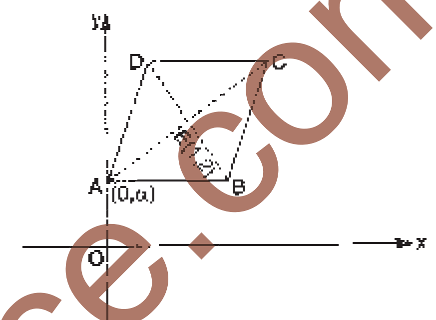
$$\tan\theta = \frac{m - 7}{1 + 7m} = -\frac{m - (-1)}{1 + m(-1)} \Rightarrow m = \frac{1}{3} \text{ or } -3.$$

Hence, the equations of third side are ;

$$y + 10 = \frac{1}{3}(x - 1) \quad \text{and} \quad y + 10 = -3(x - 1)$$

i.e., $\quad x - 3y - 31 = 0 \quad$ and $\quad 3x + y + 7 = 0$

7. Let the co-ordinates of A be $(0, \alpha)$ since, the sides AB and AD are parallel to the lines $y = x + 2$ and $y = 7x + 3$ respectively.



$\therefore$ The diagonal AC is parallel to the bisector of the angle between these two lines. The equation of the bisectors are given by,

$$\frac{x - y + 2}{\sqrt{2}} = \pm\frac{7x - y + 3}{\sqrt{50}}$$

$$\Rightarrow \quad 5(x - y + 2) = \pm(7x - y + 3)$$

$$\Rightarrow \quad 2x + 4y - 7 = 0 \quad \text{and} \quad 12x - 6y + 13 = 0.$$

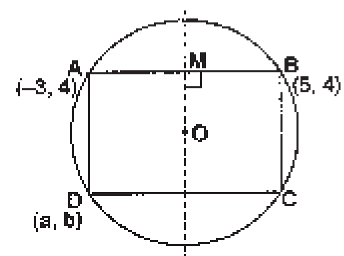
Thus, the diagonals of the rhombus are parallel to the lines $2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$.

$$\therefore \quad \text{Slope of } AE = -\frac{2}{4} \quad \text{or} \quad \frac{-12}{-6}$$

$$\Rightarrow \quad \frac{2 - \alpha}{1 - 0} = -\frac{1}{2} \quad \text{or} \quad \frac{2 - \alpha}{1 - 0} = 2$$

$\Rightarrow \alpha = \frac{5}{2}$ or $\alpha = 0$. Hence, the co-ordinates are $(0, 5/2)$ and $(0, 0)$.

8. Let O be the centre of circle M be mid point of AB.



Then, $\quad OM \perp AB \quad \Rightarrow \quad M(1, 4)$

Since, slope of $AB = 0$

Equation of straight line MO is $x = 1$ and equation of diameter is $4y = x + 7$

$\Rightarrow \qquad$ Centre is $(1, 2)$.

Also O is mid point of BD

$$\Rightarrow \quad \left(\frac{\alpha+5}{2}, \frac{\beta+4}{2}\right) = (1, 2)$$

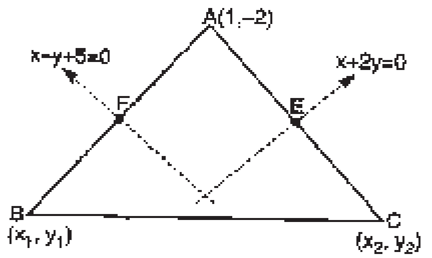
$$\Rightarrow \quad \alpha = -3, \beta = 0$$

$$\therefore \quad AD = \sqrt{(-3+3)^2 + (4-0)^2} = 4$$

$$AB = \sqrt{64+0} = 8$$

Thus, area of rectangle $= 8 \times 4 = 32$ sq. units.

9. Let the co-ordinates of B and C be (x_1, y_1) and (x_2, y_2) respectively. Let m_1 and m_2 be the slopes of AB and AC respectively. Then



$$m_1 = \text{slope of } AB = \frac{y_1+2}{x_1-1}$$

$$m_2 = \text{slope of } AC = \frac{y_2+2}{x_2-1}$$

Let F and E be the mid-point of AB and AC respectively. Then, the co-ordinates of E and F are

$$E\left(\frac{x_2+1}{2}, \frac{y_2-2}{2}\right) \text{ and } F\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right) \text{respectively.}$$

Now, F lies on $x - y + 5 = 0$.

$$\Rightarrow \quad \frac{x_1+1}{2} - \frac{y_1-2}{2} = -5$$

$$\Rightarrow \quad x_1 - y_1 + 13 = 0 \qquad \ldots(1)$$

Since, AB is perpendicular to $x - y + 5 = 0$

$\therefore$ (slope of AB) (slope of $x - y + 5 = 0$) $= -1$.

$$\Rightarrow \quad \frac{y_1+2}{x_1-1} \cdot (1) = -1$$

$$\Rightarrow \quad y_1 + 2 = -x_1 + 1$$

$$\Rightarrow \quad x_1 + y_1 + 1 = 0 \qquad \ldots(2)$$

Solving (1) and (2), we get

$x_1 = -7$, $y_1 = 6$. So, the co-ordinates of B are $(-7, 6)$.

Now, E lies on $x + 2y = 0$

$$\therefore \quad \frac{x_2+1}{2} + (y_2 - 2) = 0$$

$$\Rightarrow \quad x_2 + 2y_2 - 3 = 0. \qquad \ldots(3)$$

Since, AC is perpendicular to $x + 2y = 0$

(slope of AC) . (slope of $x + 2y = 0$) $= -1$.

$$\Rightarrow \quad \frac{y_2+2}{x_2-1} \cdot \left(-\frac{1}{2}\right) = -1$$

$$\Rightarrow \quad 2x_2 - y_2 = 4 \qquad \ldots(4)$$

Solving (3) and (4), we get

$$x_2 = \frac{11}{5} \quad \text{and} \quad y_2 = \frac{2}{5}$$

So, the co-ordinates of C are $\left(\frac{11}{5}, \frac{2}{5}\right)$.

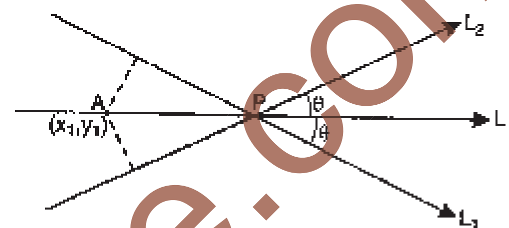
Thus, the equation of BC is

$$y - 6 = \frac{2/5 - 6}{11/5 + 7}(x+7)$$

$$\Rightarrow \quad -23(y-6) = 14(x+7)$$

$$\Rightarrow \quad 14x + 23y - 40 = 0.$$

10. Since, the required line L passes through the intersection of $L_1 = 0$ and $L_2 = 0$.



So, the equation of the required line L is

$L_1 + \lambda L_2 = 0$ *i.e.*,

$$(ax + by + c) + \lambda(lx + my + n) = 0 \qquad \ldots(i)$$

where λ is parameter.

Since, L_1 is the angle bisector of $L = 0$ and $L_2 = 0$.

$\therefore$ any point $A(x_1, y_1)$ on L_1 is equidistant from $L = 0$ and $L_2 = 0$.

$$\Rightarrow \quad \frac{|lx_1 + my_1 + n|}{\sqrt{l^2 + m^2}} = \frac{|(ax_1 + by_1 + c) + \lambda(lx_1 - my_1 + n)|}{\sqrt{(a+\lambda l)^2 + (b+\lambda m)^2}} \qquad \ldots(ii)$$

But $A(x_1, y_1)$ lies on L_1. So, it must satisfy the equation of L_1, *i.e.*, $ax_1 + by_1 + c_1 = 0$.

Substituting $ax_1 + by_1 + c = 0$ in (ii), we get

$$\frac{|lx_1 + my_1 + n|}{\sqrt{l^2 + m^2}} = \frac{|0 + \lambda(lx_1 + my_1 + n)|}{\sqrt{(a+\lambda l)^2 + (b+\lambda m)^2}}$$

$$\Rightarrow \quad \lambda^2(l^2 + m^2) = (a + \lambda l)^2 + (b + \lambda m)^2$$

$$\therefore \quad \lambda = -\frac{(a^2 + b^2)}{2(al + bm)}$$

Substituting the value of λ in (i), we get

$$(ax + by + c) - \frac{(a^2 + b^2)}{2(al + bm)}(lx + my + n) = 0$$

or $2(al + bm)(ax + by + c) - (a^2 + b^2)(lx + my + n) = 0$

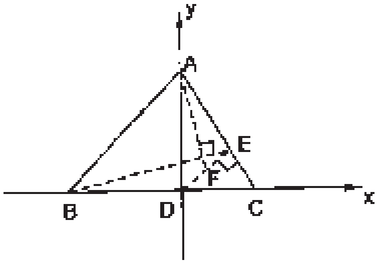
as the required equation of line L.

11. Let BC be taken as x-axis with origin at D, the mid-point of BC, and DA will be y-axis

$$AB = AC$$

Let $BC = 2a$, then the coordinates of B and C are $(-a, 0)$ and $(a, 0)$ let $\quad A(0, h)$

Then, equation of AC is,



$$\frac{x}{a}+\frac{y}{h}=1 \qquad ...(1)$$

and equation of $DE \perp AC$ and passing through origin is,

$$\frac{x}{h}-\frac{y}{a}=0 \Rightarrow x=\frac{hy}{a} \qquad ...(2)$$

Solving (1) and (2) we get the coordinates of pt E as follows :

$$\frac{hy}{a^2}+\frac{y}{h}=1 \quad \Rightarrow \quad y=\frac{a^2h}{a^2+h^2}$$

$$\therefore \quad E=\left(\frac{ah^2}{a^2+h^2},\frac{a^2h}{a^2+h^2}\right)$$

Since F is mid-point of DE,

$$\therefore \quad F\left(\frac{ah^2}{2(a^2+h^2)},\frac{a^2h}{2(a^2+h^2)}\right)$$

$$\therefore \text{ slope of } AF=\frac{h-\frac{a^2h}{2(a^2+h^2)}}{0-\frac{ah^2}{2(a^2+h^2)}}=\frac{2h(a^2+h^2)-a^2h}{-ah^2}$$

$$\Rightarrow \quad m_1=\frac{-(a^2+2h^2)}{ah} \qquad ...(3)$$

$$\text{and slope of } BE=\frac{\frac{a^2h}{a^2+h^2}-0}{\frac{ah^2}{a^2+h^2}+a}=\frac{a^2h}{ah^2+a^3+ah^2}$$

$$\Rightarrow \quad m_2=\frac{ah}{a^2+2h^2} \qquad ...(4)$$

from (3) and (4),

$$m_1m_2=-1$$

$$\Rightarrow \quad AF \perp BE.$$

12. The equation of the line AB is

$$\frac{x}{7}+\frac{y}{-5}=1 \qquad ...(1)$$

or $\qquad 5x-7y=35.$

Equation of line perpendicular to AB is,

$$7x+5y=\lambda \qquad ...(2)$$

It meets x-axis $P(\lambda/7, 0)$ and y-axis $Q(0, \lambda/5)$

The equations of lines AQ and BP are $\frac{x}{7}+\frac{5y}{\lambda}=1$ and $\frac{7x}{\lambda}-\frac{y}{5}=1$ respectively.

Let $Q(h, k)$ be their point of intersection

Then, $\frac{h}{7}+\frac{5k}{\lambda}=1$ and $\frac{7h}{\lambda}-\frac{k}{5}=1$

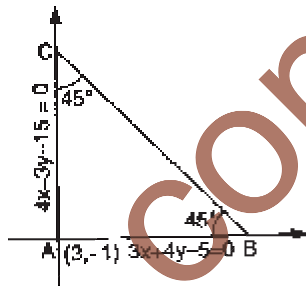
$$\Rightarrow \quad \frac{1}{5k}\left(1-\frac{h}{7}\right)=\frac{1}{7h}\left(1-\frac{k}{5}\right) \text{ \{on eliminating } \lambda\text{\}}$$

$$\Rightarrow \quad h(7-h)=k(5+k)$$

$$\Rightarrow \quad h^2+k^2-7h+5k=0$$

Hence, the locus is $x^2+y^2-7x+5y=0.$

13. Let m_1 and m_2 be the slopes of the lines $3x+4y=5$ and $4x-3y=15$ respectively.



Then, $m_1=-\frac{3}{4}$ and $m_2=\frac{4}{3}$. Clearly, $m_1m_2=-1$. So, lines AB and AC are at right angle. Thus, the triangle ABC is a right angled isosceles Δ. Hence, the line BC through (1, 2) will make an angle of 45° with the given lines. So, the possible equations of BC are

$$(y-2)=\frac{m\pm\tan 45°}{1\mp m\tan 45°}(x-1)$$

where, m = slope of $AB=-\frac{3}{4}$

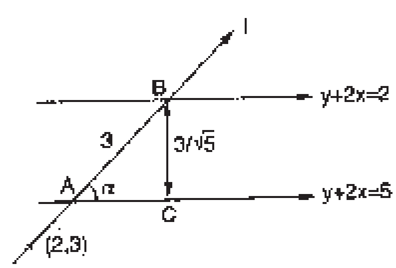
$$\Rightarrow \quad (y-2)=\frac{-\frac{3}{4}\pm 1}{1\mp\left(-\frac{3}{4}\right)}(x-1).$$

$$\Rightarrow \quad (y-2)=\frac{-3\pm 4}{4\pm 3}(x-1)$$

$$\Rightarrow (y-2)=\frac{1}{7}(x-1) \quad \text{and} \quad (y-2)=-7(x-1)$$

$$\Rightarrow \quad x-7y+13=0 \quad \text{and} \quad 7x+y-9=0.$$

14.

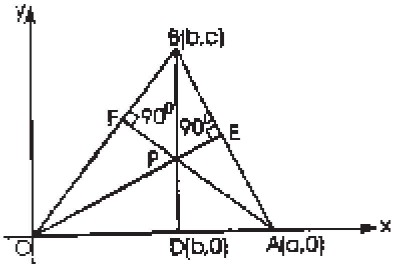


Let l makes an angle α with the given parallel lines and intercept AB is of 3 units.

As, distance between || lines

$$\Rightarrow \quad \frac{|5-2|}{\sqrt{1^2+2^2}}=\frac{3}{\sqrt{5}}$$

Now, the equation of altitude AF is



$$y-0=-\frac{b}{c}(x-a)$$

Suppose BD and OE intersect at P.
Coordinates of P are $[b, b(a-b)/c]$

Let m_1 be the slope of $OP=\frac{a-b}{c}$

and $\quad m_2=$ slope of $AB=\frac{c}{b-a}$

as $\quad m_1m_2=\left(\frac{a-b}{c}\right)\left(\frac{c}{b-a}\right)=-1$

we get that the line through O and P is $\perp$ to AB.

20. [**Imp. Note :** $d:(P,Q)=|x_1-x_2|+|y_1-y_2|$. It is new method of representing distance between two points P and Q and in future very important in coordinate geometry.]

Now let $P(x, y)$ be any pont in the first quadrant. We have

$d(P,0)=|x-0|+|y-0|=|x|+|y|=x+y\ (\because x, y>0)$

$d(P,A)=|X-3|+|Y-2|$ (given)

$d(P,0)=D(P,A)$ (given)

$\Rightarrow \quad x+y=|x-3|+|y-2|$ ...(1)

Case 1. $\quad 0<x<3, 0<y<2$

In this case (1) becomes

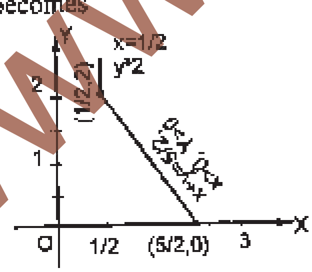
$$x+y=3-x+2-y$$

$\Rightarrow \quad 2x+2y=5$

or $\quad x+y=5/2$

Case 2. $\quad 0<x<3, y\geq 2$

Now, (1) becomes



$$x+y=3-x+y-2$$

6 Assertion And Reason

1. In ΔOPQ

Clearly, $\quad \frac{PR}{RQ}=\frac{OP}{OQ}=\frac{2\sqrt{2}}{\sqrt{5}}.$

$\Rightarrow \quad 2x=1$

$\Rightarrow \quad x=1/2$

Case 3. $\quad x\geq 3, 0<y<2$

Now, (1) becomes

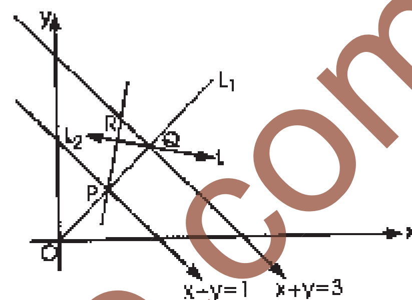
$$x+y=x-3+2-y$$

$\Rightarrow \quad 2y=-1 \quad$ or $\quad y=-1/2$

Hence no solution.

Case 4. $\quad x\geq 3, y\geq 2$

In this case (1) changes to



$$x+y=x-3+y-2$$

$\Rightarrow \quad 0=-5$

which is not possible

Hence, the solution set is

$$\{(x, y)\mid x=1/2, y\geq 2\}\cup\{(x, y)\mid x+y=5/2, 0<x<3, 0<y>2\}$$

The graph is given in adjoining figure.

21. Let the equation of straight line L be

$$y=mx$$

$$P\equiv\left(\frac{1}{m+1}, \frac{m}{m+1}\right)$$

$$Q\equiv\left(\frac{3}{m+1}, \frac{3m}{m+1}\right)$$

Now equation of $L_1: y-2x=\frac{m-2}{m+1}$ ...(1)

equation of $L_2: y+3x=\frac{3m+9}{m+1}$ ...(2)

By eliminating 'm' from equation (1) and (2), we get locus of R as $x-3y+5=0$, which represents a straight line.