

Mechanical Properties of Solid & Fluids

Subjective Problem

Q 1.

A column of mercury of 10 cm length is contained in the middle of a narrow horizontal 1 m long tube which is closed at both the ends. Both the halves of the tube contain air at a pressure of 76 cm of mercury. By what distance will the column of mercury be displaced if the tube is held vertically? (IIT JEE 1978 – 4 Marks)

Q 2.

A point mass m is suspended at the end of a mass less wire of length l and cross section A . If Y is the Young's modulus for the wire, obtain the frequency of oscillation for the simple harmonic motion along the vertical line. (IIT JEE 1978 – 4 Marks)

Q 3.

A cube of wood supporting 200 gm mass just floats in water. When the mass is removed. The cube rises by

2 cm. What is the size of the cube? (IIT JEE 1978 – 4 Marks)

Q 4.

A beaker containing water is placed on the pan of balance which shows a reading of M gms. A lump of sugar of mass m gms and volume V_{cc} . Is now suspended by a thread in such a way that it is completely immersed in water without touching the beaker and without any overflow of water. What will be the reading of the balance just when the lump of sugar is immersed? How will the reading change as the time passes on?

(IIT JEE 1978 – 4 Marks)

Q 5.

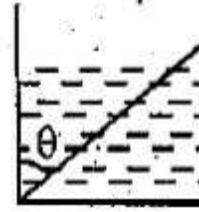
A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level? (IIT JEE 1979 – 4 Marks)

Q 6.

Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ (ρ). The height of the liquid in one vessel is h_1 and in other is h_2 . The area of either base is A . What is the work done by gravity in equalizing the levels when the two vessels are connected? (IIT JEE 1981 - 4 Marks)

Q 7.

A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in fig. The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the horizontal in the equilibrium position. (Exclude the case $\theta = 0^\circ$)



(IIT JEE 1984 – 8 Marks)

Q 8.

A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1 . Next, the ball is released and it falls through the same height before striking the surface of a liquid of density of d_L .

(IIT JEE 1992 – 8 Marks)

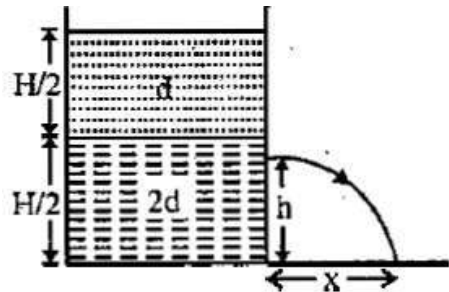
If $d < d_L$, obtain an expression (in terms of d , t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.

Is the motion of the ball simple harmonic?

If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

Q 9.

A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $H/2$ as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0 .



(IIT JEE 1995 – 5 + 5 Marks)

A homogeneous solid cylinder of length L ($L < H/2$), cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid $L/4$ in the denser liquid. Determine.

The density D of the solid and

The total pressure at the bottom of the container.

The cylinder is removed and the original arrangement is restored. A tiny hole of area s ($s \ll A$) is punched on the vertical side of the container at a height h ($h < H/2$). Determine :

The initial speed of efflux of the liquid at the hole, \

the horizontal distance x travelled by the liquid initially, and

the height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m

(Neglect the air resistance in these calculations.)

Q 10.

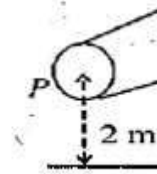
A large open top container of negligible mass and uniform cross-sectional area A has small holes of cross-sectional area $A / 100$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0 . Assuming that the liquid starts flowing out horizontally through the hole at $t = 0$, Calculate

(IIT JEE 1997 C -5 Marks)

- (i) the acceleration of the container, and
- (ii) its velocity when 75% of the liquid has drained out.

Q 11.

A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in Figure. The area of cross section of the tube two points P and Q at height of 2 metres and 5 metres are respectively $4 \times 10^{-3} \text{ m}^2$ and $8 \times 10^{-3} \text{ m}^2$. The velocity of the liquid at point P is 1 m/s. Find the work



done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q

(IIT JEE -1997 – 5 Marks)

Q 12.

A wooden stick of length L , radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in liquid of density σ ($> \rho$).

(IIT JEE 1999 – 10 Marks)

Q 13.

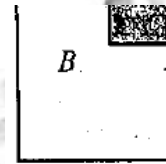
A uniform solid cylinder of density 0.8 g / cm^3 floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of the liquids A and B are 0.7 g/cm^3 and 1.2 g/cm^3 , respectively. The height of liquid A is $h_A = 1.2 \text{ cm}$. The length of the part of the cylinder immersed in liquid B

$h_B = 0.8 \text{ cm}$.

Find the total force exerted by liquid A on the cylinder.

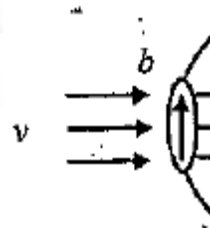
Find h , the length of the part of the cylinder in air.

The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released. **(IIT JEE 2002 – 5 Marks)**



Q 14.

A bubble having surface tension T and radius R is formed on a ring radius v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring.



(IIT JEE 2003 – 4 Marks)

Q 15.

Shown in the figure is a container whose top and bottom diameters are D and d respectively. At the bottom of the container, there is a capillary tube of outer radius b and inner radius a .

The volume flow rate in the capillary is Q . If the capillary is removed the liquid comes out with a velocity of v_0 . The density of the liquid is given as ρ . Calculate the coefficient of viscosity η .



(IIT JEE 2003 – 4 Marks)

Q 16.

A tube has two area of cross- sections as shown in figure. The diameters of the tube are 8 mm and 2 mm. Find range of water falling on horizontal surface, if piston is moving with a constant velocity of 0.25 m/s, $h = 1.25$ m ($g = 10 \text{ m/s}^2$)



(IIT JEE 2004 – 2 Marks)

Q 17.

A uniform wire having mass per unit length λ is placed over a liquid surface. The wire causes the liquid to depress by y ($y \ll a$) as shown in figure. Find surface tension of liquid. Neglect end effect.



(IIT JEE 2004 – 2 Marks)

Q 18.

A U tube is rotated about one of it's limbs with an angular velocity ω . Find the difference in height H of the liquid (density ρ) level, where diameter of the tube $d \ll L$



(IIT JEE 2005 – 2 marks)

Mechanical Properties of Solids & Fluids - Solutions

Subjective Problems

Sol 1.

M is the mid-point of tube AB.

At equilibrium

$$p_1 \times A + mg = p_2 \times A$$

$$p_1 \times A + 10 \times A \times d_{Hg} = p_2 \times A$$

$$\Rightarrow p_1 + 10d_{Hg} \times g = p_2$$

For air present in column AP

$$p \times 45 \times A = p_1 \times (45 + x) \times A$$

$$\Rightarrow p_1 = 45 / 45 + x \times 76 d_{Hg} \times g$$

.....(ii)

For air present in column QB

$$p \times 45 \times A = p_2 \times (45 - x) \times A$$

$$\Rightarrow p_2 = 45 / 45 - x \times 76 d_{Hg} \times g \quad \text{(iii)}$$

From (i), (ii) and (iii)

$$45 \times 76 \times d_{Hg} \times g / 45 + x + 10 d_{Hg} \times g = 45 / 45 - x \times 76 \times d_{Hg} \times g$$

$$\Rightarrow 45 \times 76 / 45 + x + 10 = 45 \times 76 / 45 - x$$

$$x = 2.95 \text{ cm.}$$

Sol 2.

From fig. (b), due to equilibrium

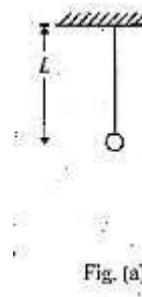
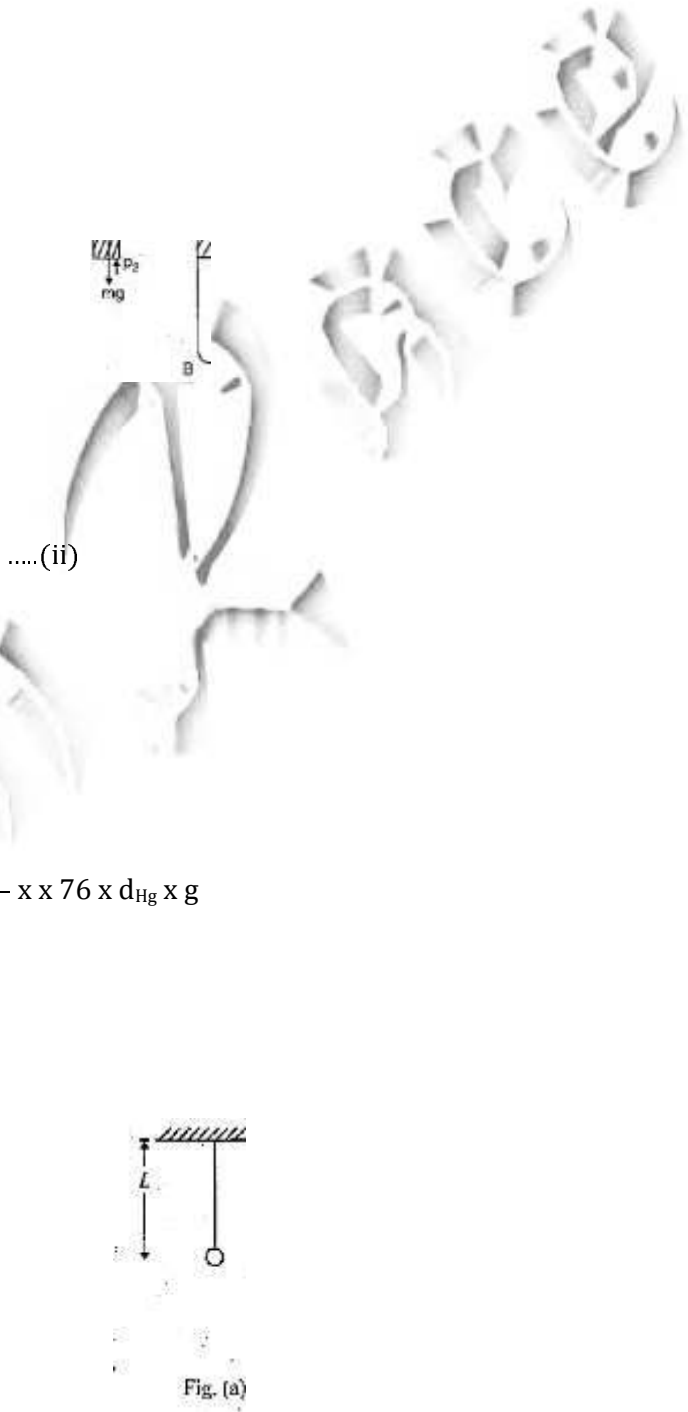
$$T = mg \quad \text{....(i)}$$

$$\text{But } Y = T / A / \ell / L$$

$$\Rightarrow T = YA \ell / L$$

From (i) and (ii)

$$mg = YA \ell / L \quad \text{(iii)}$$



From fig. (c)

Restoring force

$$= - [T - mg] = - [YA (\ell + x) / L - YA \ell / L] \text{ [from (iii)]}$$

$$= - YA x / L$$

On comparing this equation with $F = - m\omega^2 x$, we get

$$m\omega^2 = YA / L$$

$$\Rightarrow \omega = \sqrt{YA / mL} \Rightarrow 2\pi / T = \sqrt{YA / mL}$$

$$\text{Frequency } f = 1 / T = 1 / 2\pi \sqrt{YA / mL}$$

Sol 3.

Let the edge of cube be ℓ . When mass is on the cube of wood

$$200 \text{ g} + \ell^3 d_{\text{wood}} g = \ell^3 d_{\text{water}} g$$

$$\Rightarrow \ell^3 d_{\text{wood}} = \ell^3 d_{\text{water}} - 200 \quad \dots \text{ (i)}$$

When the mass is removed

$$\ell^3 d_{\text{wood}} = (\ell - 2) \ell^2 d_{\text{water}} \quad \dots \text{ (ii)}$$

From (i) and (ii)

$$\ell^3 d_{\text{water}} - 200 = (\ell - 2) \ell^2 d_{\text{water}}$$

But $d_{\text{water}} = 1$

$$\therefore \ell^3 - 200 = \ell^2 (\ell - 2)$$

$$\Rightarrow \ell = 10 \text{ cm}$$

Sol 4. When the lump of sugar is just immersed

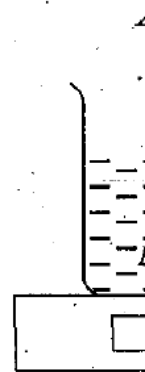
$$T = Mg - B$$

(For equilibrium of lump of sugar)

The reading on the pan balance

$$= Mg + Vd_w g$$

Where V = Volume of lump of sugar



d_w = density of water

When the lump is half dissolved,

The reading on the pan balance = $Mg + v / 2 d_s g + V / 2 d_w g$

When d_s = density of sugar

Since $d_s > d_w$

\therefore the reading will increase.

Thus, we can conclude that as the time passes the reading will keep increasing.

Sol 5.

When the stones were in the boat, the weight of stones were balanced by the buoyant force.

$$V_s d_s = V_\ell d_\ell$$

V_ℓ, V_s = volue of liquid and stone respectively

V_ℓ, V_s = density of liquid and stone respectively

Since, $d_s > d_\ell \quad \therefore V_s < V_\ell$

Therefore when stones are put in water, the level of water falls.

Sol 6.

P.E. of liquid in cylinder 1

$$U_1 = (m) g h_1 / 2 = (\rho \times A \times h_1) g h_1 / 2 = \rho A g h_1^2 / 2$$

Note :

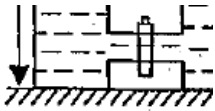
[The total mass can be supposed to be concentrated at the center of the filled part which will be at height $h_1 / 2$]

Similarly P.E. of liquid in cylinder 2 $U_2 = \rho A g h_2^2 / 2$

\therefore Total P.E. initially $U = U_1 + U_2 (h_1^2 + h_2^2)$

After the equalising of levels.

P.E. of liquid in cylinder 1 $U_1 = m g h / 2 = \rho A g / 2 h^2$



P.E. of liquid in cylinder $2 U_2' = m g h / 2 = \rho A g / 2 h^2$

\therefore Total P.E. finally $U' = U_1' + U_2' = \rho A g h^2$

The change in P.E.

$$\Delta U = U - U' = \rho A g [h^2 / 2 + h^2 / 2 - h^2]$$

Total volume remains the same.

$$A h_1 + A h_2 = A h + A h$$

$$\Rightarrow h = h_1 + h_2 / 2$$

Therefore,

$$\Delta U = \rho A g [h^2 / 2 + h^2 / 2 - (h_1 + h_2 / 2)^2]$$

$$= \rho A g / 4 (h_1 - h_2)^2$$

This change in P.E. is the work done by gravity

Sol 7. (a) For equilibrium $F_{\text{net.}} = 0$ and $\tau_{\text{net}} = 0$

Taking moment about O

$$m g \times \ell / 2 \sin \theta = F_T (\ell - x / 2) \sin \theta \quad \dots (i)$$

$$\text{Also } F_T = \text{wt. of fluid displaced} = [(\ell - x)] \times \rho_w g \quad \dots (ii)$$

$$\text{And } m = (\ell A) 0.5 \rho_w \quad \dots (iii)$$

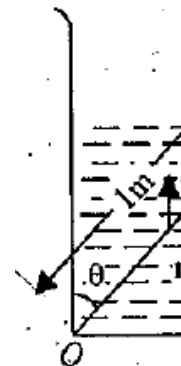
Where A is the area of cross section of the rod

From (i), (ii) and (iii)

$$(\ell A) 0.5 \rho_w g \times \ell / 2 \sin \theta = [(\ell - x) A] \rho_w g \times (\ell - x / 2) \sin \theta$$

$$\text{Here, } \ell = 1 \text{ m } \therefore (1 - x)^2 = 0.5 \Rightarrow x = 0.293 \text{ m}$$

$$\text{From the diagram } \cos \theta = 0.5 / 1 - x = 0.5 / 0.707 \Rightarrow \theta = 45^\circ$$



Sol 8.

(a) Let the ball be dropped from a height h . During fall

$$V = ut + at = 0 + g t_1 / 2 \Rightarrow t_1 = 2v / g$$

In the second case the ball is made to fall through the same height and then the ball strikes the surface of liquid of density d_L . When the ball reaches inside the liquid, it is under the influence of two force (i) Vdg , the weight of ball in downward direction (ii) Vd_Lg , the upthrust in upward direction.

Note :

The viscous forces are absent

(given)

Since, $d_L > d$

The upward force is greater and the ball starts retarding.

For motion B to C

$$u = V, v = 0, t = t, a = -a$$

$$v = u + at \Rightarrow 0 = v + (-a) t$$

$$\Rightarrow t = v / a$$

$$\text{Now, } a = F_{\text{net}} / m = Vd_Lg - Vdg / Vd = (d_L - d)g / d$$

$$\Rightarrow t = vd / (d_L - d)g \quad \dots\text{(iii)}$$

Therefore,

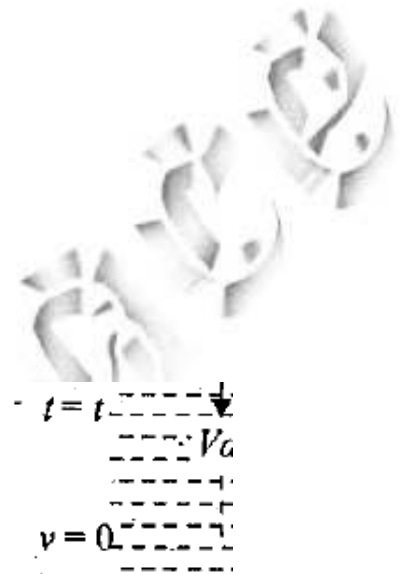
$$t_2 = t_1 + 2t = t_1 + 2vd / (d_L - d)g$$

$$= t_1 + 2d / (d_L - d)gt_1 / 2 = t_1 [1 + d / d_L - d]$$

$$\Rightarrow t_2 = d_L t_1 / d_L - d$$

(b) Since the retardation is not proportional to displacement, the motion of the ball is not simple harmonic

(c) If $d = d_L$ then the retardation $\alpha = 0$. Since the ball strikes the water surface with some velocity, it will continue with the same velocity in downward direction (until it is interrupted by some other force).



Sol 9.

(a) (i)

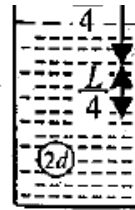
Since the cylinder is in equilibrium in the liquid therefore Weight of cylinder = upthrust

$$mg = F_{T_1} + F_{T_2} \text{ where}$$

F_{T_1} and F_{T_2} = upthrust due to lower and upper liquid respectively

$$A / 5 \times L \times D \times g = A / 5 \times L / 4 \times 2d \times g + A / 5 \times 3L / 4 \times d \times g$$

$$\Rightarrow D = 2d / 4 + 3d / 4 = 5d / 4$$



(ii) Total pressure at the bottom of the cylinder = Atmospheric pressure + Pressure due to liquid of density d + Pressure due to liquid of density 2d + Pressure due to cylinder [Weight / Area]

$$P = P_0 + H / 2 dg + H / 2 \times 2d \times g + A/5 \times L \times D \times g / A$$

$$P = P_0 + (3H / 2 + L / 4)dg \quad [\because D = 5d/4]$$

(b)

Applying Bernoulli's theorem

$$P_0 + [H / 2 \times d \times g + (H / 2 - h) 2d \times g]$$

$$= P_0 + \frac{1}{2} (2d)v^2$$

$$\Rightarrow v = \sqrt{(3H - 4h) / 4g}$$

Horizontal Distance x

$$U_x = v; t = t; \quad x = vt \quad \dots (i)$$

For vertical motion of liquid falling from hole

$$u_y = 0, S_y = h, a_y = g, t_y = t$$

$$S = ut + \frac{1}{2} at^2$$

$$\Rightarrow h = \frac{1}{2} gt^2 \Rightarrow t = \sqrt{2h / g} \dots (ii)$$

From (i) and (ii)

$$x = v_y \times \sqrt{2h / g} = \sqrt{(3H - 4H) g / 2} \times \sqrt{2h / g}$$

Reference level for P.E -

$$= \sqrt{(3H - 4h)h} \quad \dots(\text{iii})$$

For finding the value of h for which x is maximum, we differentiate equation (iii) w.r.t. t

$$\frac{dx}{dt} = \frac{1}{2} [3H - 4h]^{-1/2} \{3H - 8h\}$$

Putting $\frac{dx}{dt} = 0$ for finding values h for maxima / minima

$$\frac{1}{2} [3H - 4h]^{-1/2} [3H - 8h] = 0$$

$$\Rightarrow h = \frac{3H}{8}$$

$$\therefore x_m = \sqrt{[3H - 4(\frac{3H}{8})] \frac{3H}{8}}$$

$$= \sqrt{12H / 8 \times 3H / 8} = \frac{6H}{8} = \frac{3H}{4}$$

Sol 10.

(i) Let at any instant of time during the flow, the height of liquid in the container is x .

The velocity of flow of liquid through small hole in the orifice by Toricelli's theorem is

$$v = \sqrt{2gx} \quad \dots(\text{i})$$

The mass of liquid flowing per second through the orifice

= $\rho \times$ volume of liquid flowing per second

$$\frac{dm}{dt} = \rho \times \sqrt{2gx} \times A / 100 \quad \dots(\text{ii})$$

Therefore, the rate of change of momentum of the system in forward direction

$$= \frac{dm}{dt} \times v = 2gx \times A \times \rho / 100 \quad (\text{from (i) and (ii)})$$

(Alternatively you may use $F = \rho av^2$)

The rate of change of momentum of the system in the backward direction

$$= \text{Force on backward direction} = m \times a$$

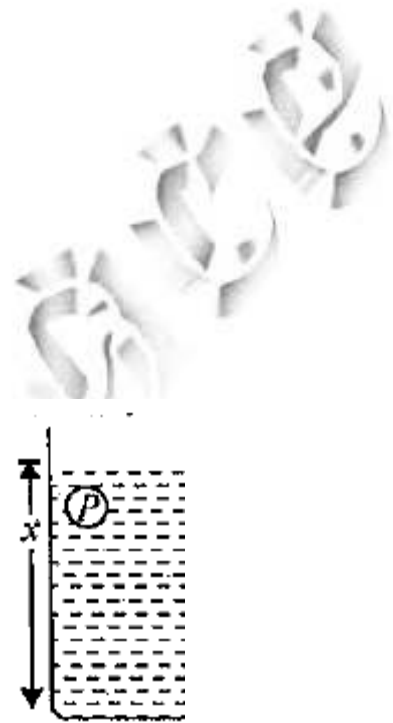
Where m is mass of liquid in the container at the instant t $m = \text{volume} \times \text{density} = A \times x \times \rho$

\therefore The rate of change of momentum of the system in the backward direction

$$A \times \rho \times x \times a$$

By conservation of linear momentum

$$A \times \rho \times x \times a = 2gx \times A \times \rho / 100 \Rightarrow a = g / 50$$



(ii) By Toricelli's theorem

$$v' = \sqrt{2g \times (0.25 h)}$$

Where h is the initial height of the liquid in the container. m_0 the initial mass is

$$m_0 = Ah \times \rho \Rightarrow h = m_0 / A\rho$$

$$\therefore v' = \sqrt{2g \times 0.25 \times m_0 / A\rho} = \sqrt{gm_0 / 2A\rho}$$

Sol 11.

Given that

$$\rho = 1000 \text{ kg/m}^3, h_1 = 2\text{m}, h_2 = 5 \text{ m}$$

$$A_1 = 4 \times 10^{-3} \text{ m}^2, A_2 = 8 \times 10^{-3} \text{ m}^2, v_1 = 1 \text{ m/s}$$

Equation of continuity

$$A_1 v_1 = A_2 v_2 \therefore v_2 = A_1 v_1 / A_2 = 0.5 \text{ m/s}$$

According to Bernoulli's theorem,

$$(p_1 - p_1) = \rho g (h_2 - h_1) - \frac{1}{2} \rho (v_2^2) - v_1^2$$

Where $(p_1 - p_2) = \text{work done / vol. [by the pressure]}$

$$\rho g (h_2 - h_1) = \text{work done/vol. [by gravity forces]}$$

Now, work done / vol. by gravity forces

$$= \rho g (h_2 - h_1) = 10^3 \times 9.8 \times 3 = 29.4 \times 10^3 \text{ J/m}^3$$

$$\text{And } \frac{1}{2} \rho (v_2^2 - v_1^2) = \frac{1}{2} \times 10^3 [1/4 - 1] = - 3/8 \times 10^3 \text{ J/m}^3$$

$$= - 0.375 \times 10^3 \text{ J/m}^3$$

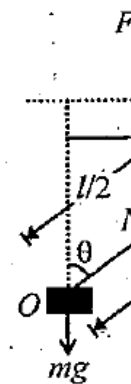
\therefore Work done / vol. by pressure

$$= 29.4 \times 10^3 - 0.375 \times 10^3 \text{ J / m}^3 = 29.025 \times 10^3 \text{ j/m}^3$$

Sol 12.

Note :

For the wooden stick-mass system to be in stable equilibrium the center of gravity of stick-mass system should be lower than the



center of buoyancy. Also in equilibrium the centre of gravity (N) and the centre of buoyancy (B) lie on the same vertical axis.

The above condition 1 will be satisfied if the mass is towards the lower side of the stick as shown in the figure. The two forces will create a torque which will bring the stick-mass system in the vertical position of the stable equilibrium.

Let ℓ be the length of the stick immersed in the liquid.

Then $OB = \ell / 2$

For vertical equilibrium

$$F_G = F_B \Rightarrow (M + m)g = F_B$$

$$\Rightarrow \pi R^2 L \rho g + mg = \pi R^2 \ell \sigma g$$

$$\ell = \pi R^2 L \rho + m / \pi R^2 \sigma \quad \dots(1)$$

Let the distance of the center of mass N of the (rod + mass) system from the origin O be $ON = y$. Then

$$y = My_1 + my_2 / M + m$$

Since mass m is at O, the origin, therefore $y_2 = 0$

$$\therefore y = M(L/2) + m \times 0 / M + m = ML / 2 (M + m)$$

$$= (\pi R^2 L \rho) L / 2 (\pi R^2 L \rho + m) \quad \dots(2)$$

Therefore for stable equilibrium

$$\ell / 2 > y$$

$$\therefore \pi R^2 L \rho + m / 2 (\pi R^2 L \sigma) > (\pi R^2 L \rho) L / 2 (\pi R^2 L \rho + m)$$

$$\Rightarrow m \geq \pi R^2 L (\sqrt{\rho \sigma} - \rho)$$

$$\therefore \text{Minimum value of } m \text{ is } \pi R^2 L (\sqrt{\rho \sigma} - \rho)$$

Sol 13.

(a) As the pressure exerted by liquid A on the cylinder is radial and symmetric, the force due to this pressure cancels out and the net value is zero.

(b) For equilibrium, Buoyant force = weight of the body

$$\Rightarrow h_A \rho_A Ag + h_B \rho_B Ag = (h_A + h + h_B) A \rho_C g$$

(where ρ_C = density of cylinder)

$$h = (h_A \rho_A + h_B \rho_B / \rho_C) - (h_A + h_B) = 0.25 \text{ cm}$$

$$(c) a = F_{\text{Buoyant}} - Mg / M$$

$$= [h_A \rho_A + \rho_B (h + h_B) - (h + h_A + h_B) \rho_C / \rho_C (h + h_A + h_C)] g$$

$$= g / 6 \text{ upwards}$$

Sol 14.

When the force due to excess pressure in the bubble equals the force of air striking at the bubble, the bubble will detach from the ring.

$$\therefore \rho A v^2 = 4T / R \times A \Rightarrow R = 4T / \rho v^2$$

Sol 15.

When the tube is not there, using Bernoulli's theorem

$$P + P_0 + \frac{1}{2} \rho v_1^2 + \rho g H = \frac{1}{2} \rho v_0^2 + P_0$$

$$\Rightarrow P + \rho g H = \frac{1}{2} \rho (v_0^2 - v_1^2)$$

But according to equation of continuity

$$Y_1 = A_2 v_0 / A_1$$

$$\therefore P + \rho g H = \frac{1}{2} \rho [v_0^2 - (A_2 / A_1 v_0)^2]$$

$$P + \rho g H = \frac{1}{2} \rho v_0^2 [1 - (A_2 / A_1)^2]$$

$$\text{Here, } P + \rho g H = \Delta P$$

According to Poiseuille's equation

$$Q = \pi (\Delta P) a^4 / 8 \eta l \Rightarrow \eta = \pi (\Delta P) a^4 / 8 Q l$$

$$\therefore \eta = \pi (P + \rho g H) a^4 / 8 Q l = \pi / 8 Q l \times \frac{1}{2} \rho v_0^2 [1 - (A_2 / A_1)^2] \times a^4$$

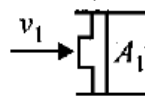
$$\text{Where } A_2 / A_1 = d^2 / D^2$$

$$H = \pi / 8 Q l \times \frac{1}{2} \rho v_0^2 [1 - d^4 / D^4] \times a^4$$

Sol 16.

From law of continuity

$$A_1 v_1 = A_2 v_2 \text{ Given } A_1 = \pi \times (4 \times 10^{-3} \text{ m})^2, A_2 = \pi \times (1 \times 10^{-3} \text{ m})^2$$



$$v_1 = 0.25 \text{ m/s}$$

$$\therefore v_2 = \pi \times (4 \times 10^{-3})^2 \times 0.25 / \pi \times (1 \times 10^{-3})^2$$

$$\text{Also, } h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{2h / g}$$

$$\text{Horizontal range } x = v^2 \sqrt{2h / g} = 4 \times \sqrt{2 \times 1.25 / 10} = 2 \text{ m}$$

Sol 17.

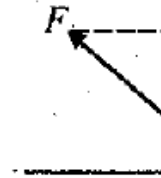
The free body diagram of wire is given below. If ℓ is the length of wire, then for equilibrium $2E \sin \theta = W$.

$$F = S \times \ell$$

$$\text{or, } 2S \times \ell \times \sin \theta = \lambda \times \ell \times g$$

$$\text{or, } S = \lambda g / 2 \sin \theta$$

$$\therefore S = \lambda g / 2y / a = a \lambda g / 2y \quad [\because \sin \theta = y / a]$$



Sol 18

Weight of liquid of height H

$$= \pi d^2 / 4 \times H \times \rho \times g \quad \dots(i)$$

Let us consider a mass dm situated at a distance x from A as shown in the figure. The centripetal force required for the mass to rotate = $(dm) \omega^2$

\therefore The total centripetal force required for the mass of length L to rotate

$$= \int_0^L (dm) \omega^2 \text{ where } dm = \rho \times \pi d^2 / 4 \times dx$$

\therefore Total centripetal force

$$= \int_0^L \left(\rho \times \frac{\pi d^2}{4} \times dx \right) \times (x \omega^2)$$

$$= \rho \times \pi d^2 / 4 \times \omega^2 \times L^2 / 2 \quad \dots(ii)$$

This centripetal force is provided by the weight of liquid of height H.

From (i) and (ii)

$$\pi d^2 / 4 \times H \times \rho \times g = \rho \times \pi d^2 / 4 \times \omega^2 \times L^2 / 2$$

$$H = \omega^2 L^2 / 2g$$

