(1) Prove that $y = A e^{x} + B e^{-2x} + x^2 + x$ is a solution of $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = 3 - 2x^2$,
(A, B are arbitrary constants)

(2) Solve $(x^2 + yx^2)\,dy + (y^2 - xy^2)\,dx = 0.$ $\left[\text{Ans: } \log\left|\dfrac{y}{cx}\right| = \dfrac{1}{x} + \dfrac{1}{y}\right]$

(3) Solve $\dfrac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ and find the particular solution subject to initial condition $x = 1,\ y = 1.$ $\left[\text{Ans: } e^{y} = e^{x} + \dfrac{x^3}{3} + c,\quad e^{y} = e^{x} + \dfrac{x^3 - 1}{3}\right]$

(4) Solve $xy\,dx - (x^2 + y^2)\,dy = 0.$ $\left[\text{Ans: } y = c\,e^{\frac{x^2}{2y^2}}\right]$

(5) Solve $x\dfrac{dy}{dx} = y + x\cos^2\dfrac{y}{x}.$ $\left[\text{Ans: } \tan\dfrac{y}{x} = \log|cx|\right]$

(6) Solve $y\,dx - x\,dy + \sqrt{x^2 - y^2}\,dx = 0.\ (x > 0).$ $\left[\text{Ans: } \sin^{-1}\dfrac{y}{x} = \log|cx|\right]$

(7) Solve $(2x - 3y + 4)\,dx + (3x - 2y + 1)\,dy = 0.$ [Ans: $(x + y - 3)^5 = c(y - x - 1)$]

(8) Solve $(2x + y)\,dx - (4x + 2y - 1)\,dy = 0.$ [Ans: $\log|10x + 5y - 2| = 10y - 5x + c$]

(9) Solve $\dfrac{dy}{dx} + 2y = x^2.$ [Ans: $4y = 2x^2 - 2x + 1 + c\,e^{-2x}$]

(10) y - intercept of tangent at any point (x, y) on a curve is $2xy^2$. Find the equation of the curve.

[Ans: $x - x^2y = cy$]

(11) The population of a country is doubled in 50 years. If the rate of increase of the population is proportional to the population, how many years will it take to become three times the original population ?

[Ans: 79 years]

(12) Prove that $2yy'' = (y')^2$ has solution $x = at + b$, $y = bt^2$, (a, b are arbitrary constants and t is a parameter).

(13) Prove that the differential equation of a family of circles having centres on Y - axis and touching X - axis is $(x^2 - y^2) \frac{dy}{dx} = 2xy$.

<u>Find the general solution and also the particular solution, when the initial conditions are given, of the following differential equations (14 to 24):</u>

(14) $2x (1 + y^2) dx - y (1 + 2x^2) dy = 0$. [Ans: $1 + y^2 = c (1 + 2x^2)$]

(15) $x \frac{dy}{dx} = y + xy + x + 1$ and $x = -1 \Rightarrow y = 2$. [Ans: $y + 1 = cxe^x$, $y + 1 = -3 x e^{x+1}$]

(16) $\frac{dy}{dx} = \sin (x + y) + \cos (x + y)$. $\left[\text{Ans}: 1 + \tan \frac{x + y}{2} = c e^x \right]$

(17) $xy (xdy + ydx) = 6y^3 dy$ and $x = 2 \Rightarrow y = 1$. [Ans: $y^2 (x^2 - 3y^2) = c$, $y^2 (x^2 - 3y^2) = 1$]

(18) $(6x^2 - 7y^2) dx - 14 xy dy = 0$. [Ans: $2x^3 - 7 xy^2 = c$]

Find the general solution and also the particular solution, when the initial conditions are given, of the following differential equations (14 to 24):

(19) $x y^2 \frac{dy}{dx} = x^3 + y^3$ and $x = 1 \Rightarrow y = 0$. [Ans: $y^3 = 3x^3 (\log x + c)$, $y^3 = 3x^3 \log x$]

(20) $(3x + 2y + 1) dx - (3x + 2y - 1) dy = 0$.

[Ans: $\log (15x + 10y - 1) + \frac{5}{2} (x - y) = c$]

(21) $\frac{dy}{dx} = \frac{2x - 5y + 3}{2x + 4y - 6}$. [Ans: $(4y - x - 3)(y + 2x - 3)^2 = c$]

(22) $\frac{dy}{dx} - y = x^2 + 3x - 2$. [Ans: $y = c e^x - (x^2 + 5x + 3)$]

(23) $\frac{dy}{dx} = y + 2 (x - 2)$. [Ans: $y = 2 - 2x + c e^x$]

(24) $(x + y)^2 \frac{dy}{dx} = 2 (x + y)^2 - 3$ $\left[\text{Ans: } \log \frac{x + y - 1}{x + y + 1} = 4x - 2y + c \right]$

(25) The mass of a boat and sailor together is 150 kg. The sailor applies force (using oars) of 70 N in the direction of motion. If the repellant force is 30 times the velocity in m / s, find the velocity of the boat at the end of t seconds. Initially, the boat is at rest.

$\left[\text{Ans: } \frac{7}{3} (1 - e^{-\frac{t}{5}}) \right]$

(26) The rate of cooling of a hot body in air is proportional to the difference between temperature of the body and temperature of air. Air temperature is 300 K and the body cools down from 370 K to 340 K in 15 minutes. When will the temperature of the body become 310 K ? [Ans: 52 minutes after it was 370 K]

(27) The population of a town was 2000 in 1960 A.D. and 5000 in 1970 A.D. and if the rate of population increase is proportional to the population present at the time, what will be the population in 2000 A.D. ? [Ans: 78125]

(28) The vertex of a parabola having axis along X-axis is (- a, 0) and its latus rectum is 4a. Prove that the differential equation is $1 - \left(\frac{dy}{dx}\right)^2 = \frac{2x}{y} \cdot \frac{dy}{dx}$.

(29) According to Newton's law of cooling, the rate of cooling of a body is proportional to the difference between the temperature of the body and temperature of air. At 20° C air temperature, the body cools down from 100° C to 60° C in 20 minutes. When will the body temperature become 30° C ? [Ans: 60 minutes after it was 100° C]

(30) The rate of melting of a piece of ice is proportional to its quantity at any moment. In 30 minutes, half of the ice has melted. Prove that 1/8th the original quantity will remain after 90 minutes.

(31) At any instant, the rate of decay of radium is proportional to its mass present. If the masses at times t_1 and t_2 are m_1 and m_2, then the time required to make the mass half of its original mass is $\frac{(t_2 - t_1)\log_e 2}{\log_e \frac{m_1}{m_2}}$.

(32) A body having mass 60 kg slides on the top of a table under a force of 54 sin 2t N. Force of friction is 60 times its velocity and initially the velocity is zero. Express velocity of the body as a function of time. $\left[\text{Ans : } \frac{9}{50}(\sin 2t - 2\cos 2t + 2e^{-t}) \right]$

(33) Obtain the differential equation of all the conics (except parabola) whose axes coincide with the co-ordinate axes. $\left[\text{Ans : } xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0 \right]$

(34) Obtain the differential equation of all the circles passing through the origin.

$$\left[\text{Ans : } (x^2 + y^2)\frac{d^2y}{dx^2} = 2\left(x\frac{dy}{dx} - y\right)\left[1 + \left(\frac{dy}{dx}\right)^2\right] \right]$$

(35) The rate of conversion of sugar to dextrose is proportional to the quantity of unconverted sugar. Initially, if sugar is 1000 g and 100 g is converted in first half an hour, then what amount is converted in first one and half hour ? [Ans: 271 g]

(36) Obtain the differential equation whose solution is $y = ax + bx^2$.

$$\left[\text{Ans : } x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0 \right]$$

Solve the following differential equations (37 to 44):

(37) $y\sqrt{1 + x^2}\, dx + x\sqrt{1 + y^2}\, dy = 0.$

$$\left[\text{Ans : } \sqrt{1 + x^2} + \sqrt{1 + y^2} = \log \frac{(1 + \sqrt{1 + x^2})(1 + \sqrt{1 + y^2})}{xy} + c \right]$$

(38) $\dfrac{x\,dx + y\,dy}{x\,dy - y\,dx} = \sqrt{\dfrac{a^2 - x^2 - y^2}{x^2 + y^2}}$ $\left[\text{Ans : } \sqrt{x^2 + y^2} = a \sin\left(\tan^{-1} \dfrac{y}{x} + c \right) \right]$

(39) $\left(\dfrac{x + y - a}{x + y - b} \right) \dfrac{dy}{dx} = \dfrac{x + y + a}{x + y + b}$ [Ans: $(b - a) \log [(x + y)^2 - ab] = 2 (x - y) + c$]

(40) $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy \dfrac{dy}{dx} = 0$

$$\left[\text{Ans : } \sqrt{1 + x^2} + \sqrt{1 + y^2} = \log \frac{1 + \sqrt{1 + x^2}}{x} + c \right]$$

(41) $\dfrac{dy}{dx} + \dfrac{x^2 + 3y^2}{3x^2 + y^2} = 0$ $\left[\text{Ans : } \log (x + y) + \dfrac{2xy}{(x + y)^2} = c \right]$

(42) $\dfrac{dy}{dx} = \dfrac{x^3 - 3xy^2}{3x^2y - y^3}$ $\left[\text{Ans: } x^2 - y^2 = c(x^2 + y^2)^2\right]$

(43) $\dfrac{dy}{dx} = \dfrac{x + 2y - 3}{2x + y - 3}$ $[\text{Ans: } x + y - 2 = c(x - y)^3]$

(44) $\dfrac{dy}{dx} = \cos bx - ay$ $\left[\text{Ans: } (a^2 + b^2)y = c(a^2 + b^2)e^{-x} + a\cos bx + b\sin bx\right]$

(45) Find differential equation for the family of curves represented by the equation $y = e^{ax^2 + bx + c}$, where a, b, c are a bitrary constants.

$$\left[\text{Ans: } y^2\frac{d^3y}{dx^3} - 3y\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + 2\left(\frac{dy}{dx}\right)^3 = 0\right]$$