MATHEMATICS

1. Which one of the following statements is always correct for arbitrary sets A, B and C?
   a. If $A \in B$ and $B \subseteq C$, then $A \in C$
   b. If $A \in B$ and $B \subseteq C$, then $A \in C$
   c. If $A \subseteq B$ and $B \in C$, then $A \in C$
   d. If $A \subseteq B$ and $B \in C$, then $A \subseteq C$

2. Let $A = \{N - \{1\} = \{k \in N : k > 1\}$,
   $X_2 = \{2k : k \in A\}$
   $X_3 = \{3k : k \in A\}$, ...
   $X_j = \{jk : k \in A\}$, ... then what is the complement of $\bigcup_{j=2}^{\infty} X_j$ in A equal to?
   a. A finite non-empty set
   b. $\varnothing$
   c. The set of primes
   d. The set of composite numbers

3. What are the four values of $\left(\frac{1}{2}+\frac{1}{2}\sqrt{-3}\right)^{3/4}$?
   a. $\left(\pm\frac{1}{\sqrt{2}}\right)(1\pm i)$
   b. $\left(\pm\frac{1}{\sqrt{2}}\right)\pm i$
   c. $\pm 1 \pm \frac{i}{\sqrt{2}}$
   d. $\pm 1 \pm \sqrt{3}i$

4. If a, b and c are three non-zero real numbers and $a \geq c$, then which one of the following is always correct?
   a. $a \geq ac$
   b. $|a|b \geq a|c$
   c. $|ab| \geq |ac|$
   d. $a|b| \geq a|c|$

5. Which one of the following is the set of all the real numbers x satisfying $||3-x|-|x+2|| = 5$?
   a. $[3, \infty[$
   b. $]-\infty, -2]$
   c. $]-\infty, -2] \cup [3, \infty[$
   d. $]-\infty, -3] \cup [2, \infty[$

6. Which one of the following is an example of a polynomial equation not having any real root?
   a. $x^3 + 3 = 0$
   b. $x^2 - x + 1 = 0$
   c. $x^4 - 5 = 0$
   d. $x^3 + 7 = 0$

7. What is the nature of the roots of the equation $(x-a)(x-b)+(x-b)(x-c)+(x-a)(x-c)=0$ where a, b, c are real numbers?
   a. Always positive
   b. Always negative
   c. Always real
   d. Always imaginary

8. If $\alpha_1, \alpha_2, \ldots \alpha_n$ are the roots of the equation $x^n + 1 = 0$, then what is $\{(1-\alpha_1)(1-\alpha_2)\ldots(1-\alpha_n)\}$ equal to?
   a. 0
   b. 1
   c. 2
   d. n

9. In the set $G = (\mathbb{R} - \{0\}) \times \mathbb{R}$, consider the binary operation 'o' given by (a, b) o (c, d) = (ac, bc + d). What is the identity in g for theis binary operation?
   a. (1, 1)
   b. (1, -1)
   c. (1, 0)
   d. (0, 1)

10. Let G be the additive group Z_8 of the residue classes modulo 8. Then what is the order of the residue class $\bar{3}$ in G?
    a. 8
    b. 6
    c. 4
    d. 2

11. On the set IN of natural numbers consider the binary relations
    $a \otimes b = a^b$
    $a \oplus b = a + b - ab$
    Which one of the following is correct?
    a. $\otimes$ is both commutative and associative
    b. $\oplus$ is both commutative and associative
    c. $\otimes$ is commutative and $\oplus$ is associative

d. $\oplus$ is commutative and $\odot$ is associative

12. What is the least number of elements in a non abelian group all of whose proper subgroups are cyclic?
a. 5
b. 6
c. 7
d. 8

13. What is the number of proper subgroups of the group $(Z, +)$ of integers?
a. 0
b. 1
c. 2
d. infinite

14. Consider the following statements:
1. Every cyclic group is abelian.
2. Every abelian group is cyclic.
3. There is at least one abelian group of order n, for every $n \in \mathbb{N}$
4. Every group of <4 is cyclic.
Which of the statement given above are correct?
a. 1, 2 and 3
b. 2, 3 and 4
c. 1, 3 and 4
d. 1 and 4 only

15. Let $G = \{a, a^2, \ldots, a^n = e\}$ be a group. What is the number of generators in G?
a. 2
b. 3
c. 4
d. 7

16. Consider the permutation: $\alpha = (1\,2\,3)(1\,4\,5)$ on the set $\{1, 2, 3, 4, 5\}$
What is the permutation α^{27}?
a. (5 4) (3 2)
b. (1 2 3)
c. 1) (5 4 1)
d. (1 3 2) (1 5 4)

17. The matrices $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} -i & 0 \\ 0 & 0 \end{bmatrix}$ from a group with respect to matrix multiplication. Which one of the following statements about this group is correct?
a. The group has no elements of order 4
b. The group has an element of order 3
c. The group is non-abelian
d. $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$ is its own inverse.

18. Suppose that H and K are subgroups of a group G with $O(H) = 10$ and $O(K) = 12$ If $H \cap K$ is non-trivial, then what is the number of elements in the complex HK?
a. 120
b. 60
c. 30
d. 22

19. Consider the ring $Z_n = \{\overline{0}, \overline{1}, \overline{2}, \ldots, \overline{n-1}\}$ of congruent modulo n classes. Under addition and multiplication modulo n, consider the following statements:
1. In Z_6, $\overline{4}$ divides $\overline{2}$
2. In Z_8, $\overline{3}$ divides $\overline{7}$
3. In Z_{15}, $\overline{9}$ divides $\overline{12}$
Which of the statements given above are correct?
a. 1 and 2 only
b. 2 and 3, only
c. 1 and 3, only
d. 1 and 2 and 3.

20. Let V be a vector space over a field F $\alpha \in R$ and $u \in V$. Which of the following statements is not correct?
a. $\alpha u = \theta \Rightarrow$ either $\alpha = 0$ or $u = \theta$
b. $|-1u| = |-1|u$ for all $u \in V$
c. $\alpha\theta = \theta$
d. $0u = \theta$

21. What is the dimension of the vector space over the field IR?
a. 1
b. Infinite
c. 2
d. 4

22. Consider the real vector space IR^3. The subspace $\{(x, y, z) \in IR^3 \mid y = x\}$ of IR^3 is generated by which one of the following?
a. $\{(1, 1, 0), (0, 0, 1)\}$
b. $\{(1, 1, 0), (1, 0, 0)\}$
c. $\{(1, 0, 0), (0, 1, 0)\}$
d. $\{(1, 0, 1), (0, 0, 1)\}$

23. Let $S = \{(1, i, 0), (2i, 1, 1), (0, 1+i, 1-i)\}$ be a subset of the complex vector space C^3 and $T = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$, be a subset of the real vector space R^3. Which one of the following statements is correct?

a. S and T are both basis
b. S is a basis but T is not a basis
c. S is not a basis but T is a basis
d. Neither S nor T is a basis

24. Consider the mappings:
1. $T: R^3 \to R^2 \ni T(x,y,z) = (x+1, y+z)$
2. $T: R^3 \to R \ni T(x,y) = xy$
3. $T: R^3 \to R^2 \ni T(x,y,z) = (|x|, 0)$

Which of the above are linear transformations?
a. 1, 2, 3
b. 1 and 2, only
c. 2 and 3, only
d. None of these.

25. Let $T: R^3 \to R^3$ be a linear transformation given by T(x, y, z) = (x/2, y/2, 0). What is the rank of T?
a. 4
b. 3
c. 2
d. 1

26. Let $T: R^3 \to R^3$ be a linear transformation given by T(x, y, z) = (x, y, 0). Then the null space is generated by which one of the following ?
a. (0, 0, 1)
b. (0, 1, 0)
c. (1, 0, 0)
d. None of these

27. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then what is A^n equal to ?
a. $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$
b. $\begin{bmatrix} 3^n & (-4)^n \\ 1 & (-1)^n \end{bmatrix}$
c. $\begin{bmatrix} 1+3n & 1-4n \\ n & 1-n \end{bmatrix}$
d. $\begin{bmatrix} 1+2n & 4n \\ 1+n & 1-2n \end{bmatrix}$

28. If $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ and $MA = A^{2m}$, $m \in N$ for some matrix M, then which one of the following is correct ?
a. $M = \begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$
b. $M = (a^2+b^2)^m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
c. $M = (a^m+b^m) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
d. $M = (a^2+b^2)^{m-1} \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

29. If $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$ then what is det (A-1) equal to ?
a. 2
b. –2
c. ½
d. –1/2

30. If A is a 2 × 2 non – singular matrix, then what is adj (adj A) equal to ?
a. A^2
b. A
c. A^{-1}
d. –A

31. If A is a 3 × 3 matrix with det A = 5 and if $B = 4A^2$, then what is det B equal to ?
a. 20
b. 100
c. 320
d. 1600

32. The lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar.
a. x – y + z = 0
b. x + y – z = 0
c. x – 2y + z = 0
d. x + 2y + z = 0

33. The intersection of two parabolic cylinders $y + 4z^2 = 1$ and $y = 4x^2$ is a space curve. What is its projection on the z – x plane?
a. Parabola
b. Circle
c. Ellipse
d. Hyperbola

34. Which one of the following lines are generators to the cone yz + 4zx + 3xy = 0?
a. x = y = z
b. x = -y = z
c. x = 2y = -z

d. $x = y = -z$

35. What is the position vector of the centre of the sphere $|\vec{r}|^2 + \vec{r}\cdot(\hat{i}+\hat{j}-\hat{k}) - 9 = 0$

a. $\hat{i}+\hat{j}-\hat{k}$

b. $1/2(\hat{i}+\hat{j}-\hat{k})$

c. $1/2(-\hat{i}-\hat{j}+\hat{k})$

d. $-\hat{i}-\hat{j}+\hat{k}$

36. The point (4, 1) undergoes the following two successive transformations:

1. Reflection about the line y = x.
2. Translation through a distance 2 unit along the positive x-axis.

What are the final coordinates of the point?

a. (4, 3)

b. (3, 4)

c. (1, 4)

d. (6, 1)

37. When does the equation $ax^2 + ay^2 + az^2 + 2ux + 2vy + 2wz + d = 0$, $(a \neq 0)$ represent a sphere ?

a. $u^2 + v^2 + w^2 - d \geq 0$

b. $u^2 + v^2 + w^2 - ad \geq 0$

c. $u^2 + v^2 + w^2 - d \leq 0$

d. $u^2 + v^2 + w^2 - ad < 0$

38. If $<l_1, m_1, n_1>, <l_2, m_2, n_2>, <l_3, m_3, n_3>$ are direction cosines of three mutually perpendicular lines then what is/are the value (s) of $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$?

a. 0

b. ±1

c. ±2

d. 3

39. The line $(p + 2q)x + (p - 3q)y = p - q$ for all values of p and q passes through which one of the following points ?

a. (3/2, 5/2)

b. (1/5, 2/5)

c. (3/5, 3/5)

d. (2/5, 3/5)

40. What is the equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a?

a. $x^2 + y^2 = 9a^2$

b. $x^2 + y^2 = 6a^2$

c. $x^2 + y^2 = 4a^2$

d. $x^2 + y^2 = a^2$

41. Under which one of the following conditions does the equation $ax^2 + by^2 + cx + cy = 0, c \neq 0$ represent a pair of straight lines ?

a. a + b = 0

b. a + c = 0

c. b + c = 0

d. a + b + c = 0

42. What does the curve $x = 3(\cos t + \sin t)$, $y = 4(\cos t - \sin t)$ represent ?

a. Ellipse

b. Parabola

c. Hyperbola

d. Circle

43. A, B, C are fixed points and P is a variable point. Sum of vectors $\vec{PA}$ and $\vec{PB}$ always passes through C. What is the ratio in which P divides AB ?

a. 1 : 2

b. 2 : 1

c. 1 : 1

d. 1 : 3

44. If $\vec{A} = 2\hat{i} + \hat{k}, \vec{B} = \hat{i} + \hat{j} + \hat{k}, \vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ and $\vec{r} \times \vec{B} = \vec{C} \times \vec{B}, \vec{r} \cdot \vec{A} = 0$, then what is $\vec{r}$?

a. $\hat{i} - 8\hat{j} + 2\hat{k}$

b. $2\hat{i} + 8\hat{j} + 4\hat{k}$

c. $-\hat{i} + 8\hat{j} + 2\hat{k}$

d. $-2\hat{i} - 6\hat{j} + 2\hat{k}$

45. If the vectors $\hat{i} - 2p\hat{j} + 3q\hat{k}$ and $\hat{i} + 2p\hat{j} - 3q\hat{k}$ are orthogonal, then what is the locus of the point (p, q) ?

a. Circle

b. Ellipse

c. Hyperbola

d. Pair of straight lines

46. For any vector $\vec{a}$, what is $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2$

a. $|\vec{a}|^2$

b. $2|\vec{a}|^2$

c. $3|\vec{a}|^2$

d. $4|\vec{a}|^2$

47. If $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such thet $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}(\vec{b})$, what is the angle which $\vec{a}$ makes with $\vec{c}$?

a. 30°
b. 60°
c. 90°
d. cannot be determined

48. What is the value of $\lim_{x\to\infty}\{\sin(1/x)+\cos(1/x)\}^x$?

a. 0
b. 1/e
c. e
d. 1

49. Let $f(x+y)=f(x).f(y)$ for all x and y, and $f(5)=-2$, $f'(0)=3$. What is the value of $f'(5)$?

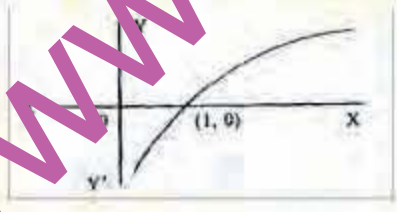
a. 3
b. 1
c. -6
d. 6

50. Which one of the following functions is well-defined?

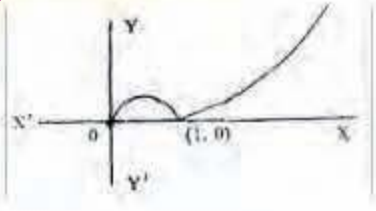
a. $\sqrt{(1+\sin x)}$
b. $\sqrt{(5\sec^2 x-4)}$
c. $\sqrt{(3-\cos^2 x)}$
d. $\sqrt{\left(x^2+x^{-2}+\frac{1}{100}\right)}$

51. Which one of the following graphs is the correct graph of the function $y = x / n x$?

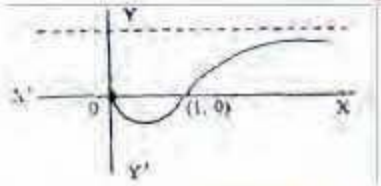
a.



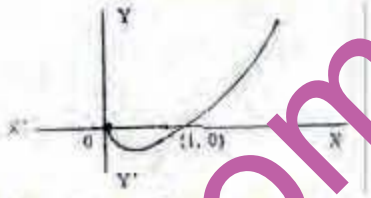
b.



c.



d.



52. The function $f(x)=\sin^3 x-m\sin x$ is defined on the open interval $(-\pi/2, \pi/2)$ and it assumes only 1 maximum value and only 1 minimum value on this interval. Then, which one of the following must be correct?

a. $0<m<3$
b. $-3<m<0$
c. $m=0$
d. $m>3$

53. What is the value of the integral $\int_0^{\pi/2}\frac{1-\sin 3x}{1+\sin x}dx$?

a. 2
b. $2\pi/3$
c. $4+\frac{3}{2}\pi$
d. $\frac{3\pi}{2}-4$

54. What is the length of an arc of the curve $y=1-\ln\cos x$ intercepted between $x=0$ and $x=\pi/4$?

a. $ln(\sqrt{2}-1)$
b. $(ln\sqrt{2}+2)$
c. $1-(ln\sqrt{2})$
d. $(ln\sqrt{2})-1$

55. Which one of the following is correct ? The function defined by $f(x)=\ln(|x-1|+2|x+1|)$ is a monotonically decreasing function on the largest interval defined by

a. $-1<x<1$
b. $x>1$
c. $x<-1$
d. $x\le -1$

56. Let $I_1=\int_0^{\pi}\frac{\left[\log\left(x+\frac{1}{x}\right)\right]^3}{(1+x^2)}$ and $I_2=\int_0^{\pi/2}\log\sin 2t\,dt$

What is the value of $(I_1 + I_2)$?

a. $\frac{\pi}{2}\log 2$

b. $\frac{-\pi\log 2}{2}$

c. $\pi\log 2$

d. $-\pi\log 2$

57. Let $\frac{e^x}{e^x+1}=\frac{1}{2}+\frac{x}{4}+\alpha x^2-\frac{x^3}{48}+\ldots$

What is the value of α?

[Use $f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\ldots$]

a. $\alpha=1/16$

b. $\alpha=1/8$

c. $\alpha=0$

d. $\alpha=-1/16$

58. Let $f(x)=\int\sec^{2/3}x\,\mathrm{cosec}^{4/3}x\,dx$. What is the value of f(x)?

a. $-3(\cot x)^{1/3}+c$

b. $-3(\tan x)^{-2/3}+c$

c. $3(\mathrm{cosec}\,x)^{1/3}+c$

d. $3(\cos 2x)^{1/3}+c$

59. Let f(x) be continuous and differentiable over (a − h, a + h), a > 0.

$$F(x)=\begin{cases}\left(\frac{f'(x)}{f(x)-f(a)}-\frac{1}{x-a}\right), & \text{if } x\neq a\\ a, & \text{if } x=a\end{cases}$$

what is F(x) continuous at x = a?

a. $\alpha=1$

b. $\alpha=f'(a)/f(a)$

c. $\alpha=a$

d. $\alpha=\frac{f'(a)}{2f(a)}$

60. Given $f(x)=\begin{cases}x, & \text{when } 0\le x<1/2\\ 1/2, & \text{when } x=1/2\\ 1-x, & \text{when } 1/2<x\le 1\end{cases}$

and $G(x)=\left(x-\frac{1}{2}\right)^2, x\in R$. What is the area in the interval $\left[\frac{1}{2},\frac{\sqrt{3}}{2}\right]$?

a. $\frac{\sqrt{3}}{4}-\frac{1}{3}$

b. $\frac{\sqrt{3}}{4}+\frac{1}{3}$

c. 0

d. $\frac{\sqrt{3}}{12}$

61. Which one of the asymptotes of the curve $y=\frac{x+3}{x+2}$?

a. y = 1 and x = -2

b. y = -1 and x = -2

c. y = 0 and x = -2

d. y = 1 and x = 2

62. Let A and B be fixed points with coordinates (0, a) and (0, b) respectively and p is a variable point (x, 0) referred to rectangular axes. When is the angle APB extremum?

a. $x=\sqrt{ab}$

b. $x^2=ab$

c. $x^2=a+b$

d. $x^2=a-b$

63. What are the values of a and b respectively, if $\lim_{x\to 0}\frac{\sin ax-\ln(e^x\cos x)}{x\sin bx}=\frac{1}{2}$?

a. 1, 1

b. $1,\frac{1}{2}$

c. $-1,1$

d. $-1,\frac{1}{2}$

64. $f(x)=\frac{\sqrt{2}\cos x-1}{\cot x-1}, x\neq\frac{\pi}{4}$

$=\alpha, x=\frac{\pi}{4}$

What should be the value of α for which f(x) is continuous at $x=\pi/4$?

a. $\frac{1}{2}$

b. $\sqrt{2}$

c. $-\frac{1}{2}$

d. 2

65. What are the order and degree respectively of the differential equation $\frac{d^2}{dx^2}\left[\left(\frac{d^2y}{dx^2}\right)^{-3/2}\right]=0$?

a. 1, 4

b. 4, 1

c. 4, 4

d. 1, 1

66. What is the solution of the differential equation $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2+y\frac{dy}{dx}=0$?

a. $y^2 = A\ln x + B$

b. $y^2 = A\ln^2 x + B$

c. $y = A\ln x + B$

d. $y = A\ln^2 x + B$

67. Which equation represents the set of all curves in the xy-plane which have slope at each point P equal to the reciprocal of the slope at each point P equal to the reciprocal of the slope of the straight line through P and the origin?

a. $y^2 = x + c$

b. $x^2 + y^2 = c^2$

c. $x^2 - y^2 = c^2$

d. $xy = c^2$

68. What is the value of the particular integral of the differential equation $(D^3 + a^2D)y = \sin ax$?

a. $-\frac{x}{2a^2}\cos ax$

b. $\frac{x}{2a^2}\sin ax$

c. $-\frac{x}{2a^2}\sin ax$

d. $\frac{x}{2a^2}\cos ax$

69. For non-exact differential equation integrating factor?

a. $1/(2x^2y^2)$

b. $1/(2x)$

c. $1/(2xy)$

d. $1/(x^2y)$

70. The differential equation $\sin 2y\frac{dy}{dx} + 2\tan\cos^2 y = 2\sec x\cos^2 y$ is reduced to the linear from $\frac{dv}{dx} + Pv = Q$, where P and Q are function of x alone. What is the substitution?

a. $\tan y = v$

b. $\sec y = v$

c. $\cot y = v$

d. $\operatorname{cosec} y = v$

71. What is the solution of the differential equation $\frac{dy}{dx} = (4x + y + 1)^2$?

a. $4x + y + 1 = 2\tan(2x + y + c)$

b. $4x + y + 1 = 2\tan(x + 2y + c)$

c. $4x + y + 1 = 2\tan(2y + c)$

d. $4x + y + 1 = 2\tan(2x + c)$

72. What is the solution of the differential equation $\frac{dy}{dx} = e^{(x-y)}\{(e^x - e^y)\}$?

a. $e^{e^y}(e^x - e^y + 1) = c$

b. $e^{e^y}(e^y - e^x + 1) = c$

c. $e^{e^x}(e^x - e^y + 1) = c$

d. $e^{e^x}(e^y - e^x + 1) = c$

73. What is the degree of the differential equation for the given curve in which $(\text{subtangent})^m = (\text{Subnormal})^n$ in Cartesian form, where $0 < n < m$, m, n, m/n are integers?

a. m + n

b. m − n

c. mn

d. m/n

74. Which one of the following curves in the orthogonal trajectory of straight lines passing through a fixed point (a, b)?

a. $x - a = k(y - b)$

b. $(x - a)(y - b) = k$

c. $(x - a)^2 = k(y - b)$

d. $(x - a)^2 + (y - b)2 = k$

75. What is the singular solution of $y^2(1 + y'^2) = r^2$ where r is a constant?

a. $y^2 = 4rx$

b. $y^2 = 4r$

c. $y^2 = r^2$

d. $y^2 = r^3$

76. A function f' is defined as follows:

$f(x) = x^{1/2}$, if $x < 1$

$= (2 - x)^{1/2}$, if $1 \le x < 2$

$= (x - 2)^{1/4}$, if $x \ge 2$

What is the total number of points where derivatives do not exist ?

a. 2

b. 2

c. 3

d. none of these

77. $f(x) = x^6 - x - 1, x \in [1, 2]$

Consider the following statements:

1. f is increasing on [1, 2]
2. f has a root in [1, 2]
3. f is decreasing on [1, 2]
4. f has no root in [1, 2]

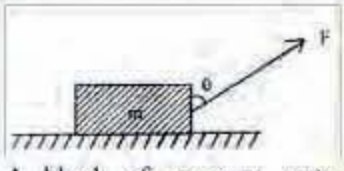
Which of the above are correct?

a. 1 and 2
b. 1 and 4
c. 2 and 3
d. 3 and 4

78. Let $f(x) = (x - a)(x - b)(x - c)$ be a real-valued function where $a < b < c (a, b, c \in R)$ such that $f''(\alpha) = 0$. Then if $\alpha \in (c_1, c_2)$, which one of the following is correct?

a. $a < c_1 < b$, and $b < c_2 < c$
b. $a < c_1, c_2 < b$
c. $b < c_1, c_2 < c$
d. None of the above (Here $c_1 \neq c_2$)

79.



A block of mass m rests on rough horizontal surface as shown in the figure given above. The coefficient of friction between the block and the surface is μ. A force $F = mg\cos\theta$ with the vertical side of the block pulls it. Under what condition can the block be pulled along the surface?

a. $\tan\theta \geq \mu$
b. $\cos\theta \geq \mu$
c. $\tan\theta \leq \mu$
d. $\cot\theta \leq \mu$

80. What is the centre of gravity of the arc of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ lying in the first quadrant?

a. (0, 0)
b. $\left(\frac{a}{2}, \frac{a}{2}\right)$
c. $\left(\frac{5a}{2}, \frac{5a}{2}\right)$
d. $\left(\frac{2a}{5}, \frac{2a}{5}\right)$

81. Forces $\vec{AB}, \vec{DC}$ act along the two sides AB and DC of a square ABCD, respectively and forces $\vec{CA}, \vec{BD}$ act along the diagonals CA, BD of the square ABCD, respectively. Which one of the following is correct?

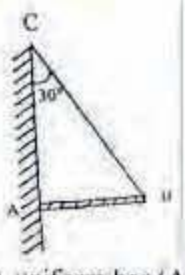
This system of forces

a. reduces to a couple
b. reduces to a single force passing through the centre of the square
c. is in equilibrium
d. reduces to a single force parallel to $\vec{AB}$

82. $\hat{i}$ and $\hat{j}$ are unit vectors along the x and y axes respectively. Coplanar forces $3\hat{i} + 4\hat{j}, 7\hat{j} - 8\hat{i}, -18\hat{i} + 7\hat{j}$, and $X\hat{i} + Y\hat{j}$, act at the points (2, 3), (3, 4), (5, 6) and (1, 2) respectively in the xy-plane. If the system is equivalent to a couple, then what are the force components X and Y?

a. X = 10, Y = 5
b. X = 8, Y = 3
c. X = 8, Y = -3
d. X = 5, Y = 3

83.



A uniform bar (AB) of weight W is hinged at A to a vertical wall and held in horizontal position by a cord (BC) as shown in the figure above. What is the tension iii the cord?

a. W
b. 2W/3
c. $\sqrt{3}$ W
d. W/ $\sqrt{3}$

84. The resultant of two forces inclined to each other at 120° is at right angle to the smaller force. If the greater force is 200 N then what are the magnitudes of the smaller force and the resultant respectively?

a. 100 N, 100 N

b. $100\sqrt{3}$ N, $100\sqrt{3}$ N

c. 100N, $100\sqrt{3}$ N

d. $100\sqrt{3}$ N, 100N

85. What is the work done by the force $\vec{F} = \frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j}$, Where V = $\tan^{-1}(x/y)$, in taking a particle from a point (0, 1) to a point (1, 0)?

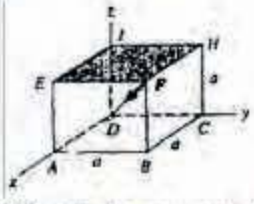
a. π

b. $\pi/2$

c. $\pi/4$

d. $\pi/6$

86.



What is the moment of 50 N force directed along the diagonal of a cube (shown above) about the corner A of the cube? (Side of the cube is 'a' m)

a. $50a\left(\hat{k}-\hat{j}\right)N-m$

b. $50a\left(\hat{j}-\hat{k}\right)N-m$

c. $\frac{50}{\sqrt{3}}a\left(\hat{k}-\hat{j}\right)N-m$

d. $\frac{50}{\sqrt{3}}a\left(\hat{j}-\hat{k}\right)N-m$

87. If a particle is moving according to the law $v2 = 2(x \sin x + \cos x)$, where v is the velocity and x is the distance described, what is its acceleration?

a. x cos x

b. x sin x

c. $\frac{x\cos x}{v}$

d. $\frac{x\sin x}{v}$

88. By what percent approximately should be velocity of moon in its orbit be increased so that it escapes?

a. 20.7%

b. 41.4%

c. 82.8%

d. 100%

89. A particle is moving on x-y plane with velocity at any instant t given $\vec{v} = v_0\hat{i} + (a\omega\cos\omega t)\hat{j}$. What is the equation to trajectory of the particle if the particle starts from origin when time t = 0?

a. $y = a\cos(\omega x/v_0)$

b. $y = a\sin(\omega x/v_0)$

c. $y = x + a\cos(\omega x/v_0)$

d. $y = x + a\sin(\omega x/v_0)$

90.

Set 1	:	get n
Comment	:	n is a natural number
Step 2	:	x ← 1
Step 3	:	p ← 1
Step 4	:	y ← 2 (2 * x -1)
Step 5	:	p ← p / y
Step 6	:	x ← x + 1
Step 7	:	If x ≤ n
		then go to step 4
		else go to step 8
Step 8	:	write p
Step 9	:	stop

What is the output of the algorithm given above?

a. $\frac{\lfloor 2n}{\lfloor n}$

b. $\frac{\lfloor 2n}{\lfloor (n)^2}$

c. $2^n \lfloor n$

d. $2^n \lfloor 2n$

91. What is the cube root of $(1000)_2^{(11)2} + (111)_2^{(11)2} + (10101000)_2 \times (1111)_2$?

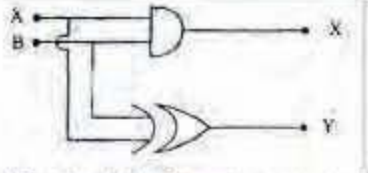
a. $(1110)_2$

b. $(10010)_2$

c. $(11011)_2$

d. $(1111)_2$

92.



If only NAND gates are to be used to realize the circuit given above, what is their minimum number among the following?

a. 8

b. 11
c. 13
d. 15

93. A ROM chip has a capacity 1024 x 4
Consider the following:
1. 4 data inputs are required
2. 4 data outputs are required
3. 1024 address inputs are required
Which of the above is/are correct ?
a. 1 only
b. 2 only
c. 1 and 2 only
d. 1, 2 and 3

94. What is the capacity of 5-M-byte memory chip?
a. 5×2^6 bits
b. 5×2^9 bits
c. 5×2^{20} bits
d. 5×2^{23} bits

95. **Assertion (A):** Two projectiles are projected with same speed making complementary angles with each other. The sum of the heights attained by them is independent of angle of projection.
Reason (R): If H_1 and H_2 are the height attained by the projectiles projected with same speed u making complementary angles with each other then $H_1 + H_2 = u^2/g$
a. Both A and R are individually true and R is the correct explanation of A
b. Both A and R are individually true but R is not the correct explanation of A
c. A is true but R is false
d. A is false but R is true

96. **Assertion (A):** A system of equations Ax = b, where A is the matrix of a linear transformation T, non-singular from a vector space, V into vector space V', has a unique solution.
Reason (R): Any T: V → V' is either 1-1 or onto.
a. Both A and R are individually true and R is the correct explanation of A
b. Both A and R are individually true but R is not the correct explanation of A
c. A is true but R is false
d. A is false but R is true

97. **Assertion (A):** In the hypothesis of Rolles' theorem, the statement that the function is continuous on a closed interval [a, b], $-\infty < a, b < \infty$ is needed to be assumed.
Reason (R): Differentiability on (a, b) does not imply continuity on [a, b].
a. Both A and R are individually true and R is the correct explanation of A
b. Both A and R are individually true but R is not the correct explanation of A
c. A is true but R is false
d. A is false but R is true

98. **Assertion (A):** $\lim_{x \to \infty} \frac{\sin x}{x} = 0$
Reason (R): $\lim_{x \to \infty} \frac{\sin x}{x} = \lim_{x \to \infty} \sin x \lim_{x \to \infty} \frac{1}{x}$
a. Both A and R are individually true and R is the correct explanation of A
b. Both A and R are individually true but R is not the correct explanation of A
c. A is true but R is false
d. A is false but R is true

99. **Assertion (A):** At the mid position between mean position and extreme position of a particle executing SHM, the kinetic energy is equal to the potential energy.
Reason (R): Both potential energy and kinetic energy depend upon position of the particle from its mean position.
a. Both A and R are individually true and R is the correct explanation of A
b. Both A and R are individually true but R is not the correct explanation of A
c. A is true but R is false
d. A is false but R is true

100. **Assertion (A):** The multiplicative group G = {1, -1, i, -i}
Reason (R): 1 is the generator of G.
a. Both A and R are individually true and R is the correct explanation of A
b. Both A and R are individually true but R is not the correct explanation of A
c. A is true but R is false
d. A is false but R is true