MATHEMATICS

1. Let $X = \{n : n \text{ is a positive integer, } n \le 50\}$. If $A = \{n \in X : n \text{ is even}\}$ and $B = \{n \in X : n \text{ is a multiple of } 7\}$, then what is the number of elements in the smallest subset of X containing both A and B?
   a. 28
   b. 29
   c. 32
   d. 35

2. Let $A = \{t \in N : 12 \text{ and } t \text{ are relatively prime}\}$ and $B = \{t \in N : t \le 24\}$. What is the number of elements in $A \cap B$?
   a. 10
   b. 8
   c. 7
   d. 4

3. Let $z = \cos(\pi/8) + i\sin(\pi/8)$ and $A = \{Z^n : n \in N\}$. which one of the following is correct.
   a. A is not a finite set
   b. A contains 12 non-real complex numbers
   c. The number of elements in A is 6
   d. A contains no integers

4. Which one of the following is correct? The equation $x^3 - 226x^2 - (26\ldots)x - (26)^3 = 0$
   a. Has no multiple roots
   b. Has exactly one real root
   c. Has no non-real roots
   d. Has no integral roots

5. What is the sum of the roots of the equation $\{(x-\ldots)^2+\ldots\}\{(x-3)^2+4\} = 0$?
   a. ...
   b. 10
   c. 3
   d. 18

6. Let m be a positive integer, $m \ge 2$. If $\alpha_1, \alpha_2, \ldots, \alpha_m$ are the roots of the equation $x^m - 1 = 0$, then what is the equation whose roots are
   $\beta_1 = \alpha_2 + \alpha_3 + \ldots + \alpha_m - (m-1)\alpha_1$
   $\beta_2 = \alpha_1 + \alpha_3 + \ldots + \alpha_m - (m-1)\alpha_2$ ⋮
   $\beta_i = \alpha_1 + \ldots + \alpha_{i-1} + \alpha_{i+1} + \ldots + \alpha_m - (m-1)\alpha_i$
   $\beta_m = \alpha_1 + \ldots \alpha_{m-1} - (m-1)\alpha_m$?
   a. $x^m + m^m = 0$
   b. $x^m - (-m)^m = 0$
   c. $x^m + (m-1)^m = 0$
   d. $x^m - (m-1)^m = 0$

7. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, then what is the value of $\sum \alpha^2\beta$?
   a. $pq + 3r$
   b. $pq - r$
   c. $pq - 3r$
   d. $q^2 - r$

8. Let G be an infinite cyclic group and H is its subgroup. Which one of the following is correct?
   a. H is not necessarily cyclic.
   b. H is finite.
   c. H is infinite.
   d. H is not necessarily abelian.

9. Let $G = \{e\}$ be a group with no subgroup other then $\{e\}$ and G. Then which one of the following is correct?
   a. G is an infinite cyclic group.
   b. G is a finite cyclic group.
   c. G is an abelian non-cyclic group.
   d. G is neither abelian nor cyclic.

10. Which one of the following group is cyclic?
    a. $Z_{12} \times Z_9$
    b. $Z_{10} \times Z_{85}$
    c. $Z_4 \times Z_{25} \times Z_6$
    d. $Z_{22} \times Z_{21} \times Z_{65}$

11. Which one of the following is a group?
    a. $(N, *)$ where $a * b = a$ for all $a, b \in N$
    b. $(Z, *)$ where $a * b = a - b$ for all $a, b \in Z$
    c. $(Q, *)$ where $a * b = ab/2$ for all $a, b \in Q$
    d. $(R, *)$ where $a * b = a + b + 1$ for all $a, b \in R$

12. Consider the group $(R^* \times R, \bullet)$, where $R^* = R - \{0\}$ and $(a,b)\bullet(c,d) = (ac, bc+d)$. What are the identity element and the inverse of (a, b) respectively?
    a. (1, 0) and (a^{-1}, ba^{-1})
    b. (0, 1) and (a^{-1}, ba^{-1})
    c. (0, 1) and $(a^{-1}, -ba^{-1})$
    d. (1, 0) and $(a^{-1}, -ba^{-1})$

13. Which one of the following statements is correct?
    a. Abelian groups may have non-abelian subgroups.
    b. Non-abelian groups may have abelian subgroups.
    c. Cyclic groups may have non-cyclic subgroups.
    d. Non-cyclic groups cannot have cyclic subgroups.

14. Let $\sigma = (1,3,5,7,11)(2,4,6) \in S_{11}$. What is the smallest positive integer n, such that $\sigma^n = \sigma^{37}$?
    a. 3
    b. 5
    c. 7
    d. 11

15. Let (R, +) be an abelian group in which multiplication (·) is defined on R by setting a.b = 0 for all a, b, ∈ R, then which one of the following statements is correct?
    a. $(R, +, \cdot)$ is not a ring.
    b. $(R, +, \cdot)$ is a ring, but not commutative.
    c. $(R, +, \cdot)$ is a commutative ring, but has no unity.
    d. $(R, +, \cdot)$ is a field.

16. Consider the following assertions
    i. The characteristic of the ring $(Z, +, \cdot)$ is zero.
    ii. For every composite number, n, Zn, the ring of residue classes modulo n, is a field.
    iii. Z5, the ring of residue classes modulo 5, is an integral domain.
    iv. The ring of all complex numbers is a field.

    Which of the above assertions are correct?
    a. i, iii and iv
    b. i, ii and iii
    c. i, ii and iv
    d. i, iii and iv

17. Let F be a finite field with n elements. What is the possible value of n?
    a. 1
    b. 36
    c. 37
    d. 125

18. If R is a finite integral domain with n element, then what is the number of invertibel elements uner multiplication in R?
    a. 1
    b. n
    c. n – 1
    d. [n/2] where [·] is the bracket function

19. If Q, R, C are respectively the fields of rational numbers, real numbers and complex numbers, then which one of the following algebraic structures is not a vector space?
    a. R over the field Q.
    b. R over the field R.
    c. C over the field R.
    d. C over the field C.

20. Let x = (3, 2, -1), y = (2, 4, 1), z = (4, 0, -3) and w = (10, 4, -5) be vectors in R3, a real vector space. Which one of the following is correct?
    a. 2x + z = w, y + z = w
    b. 2x – y = z, y + 2z = w
    c. x + z = w, 2x + y = z
    d. y + 2z = w, x – y = z

21. If V is the real vector space of all mappings from R to R, $V_1 = \{f \in V \mid f(-x) = f(x)\}$ and $V_2 = \{f \in V \mid f(-x) = f(x)\}$, then which one of the following is correct?
    a. Neither V_1 and V_2 is a subsapce of V.
    b. V_1 is a subspace of V, but V_2 is not a subsapace of V.
    c. V_1 is not a subspace of V, but V_2 is a subspace of V.
    d. Both V_1 and V_2 are subsapces of V.

22. Let F[x] be the ring of polynomials in one variable x over a field F with the relation $x^n = 0$, for a fixed $n \in N$. What is the dimension of F[x] over F?
    a. 1
    b. n – 1
    c. n
    d. infinite

23. Which one of the following is correct ? The set $S = \{a + ib, c + id\}$ is a basis for the vector space C over R iff
    a. $ad - bc = 0$
    b. $ad + bc = 0$
    c. $ad + bc \neq 0$
    d. $ad - bc \neq 0$

24. Let V be the vector space of all 2×2 matrices over the field R of real numbers and $B=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$. If $V \to V$ is a linear transformation defined by $T(A) = AB - BA$, then what is the dimension of the kernel of T?
    a. 1
    b. 2
    c. 3
    d. 4

25. What is the rank of the linear transformation $T: R^3 \to R^3$ defined by $T(x, y, z) = (y, 0, z)$?
    a. 3
    b. 2
    c. 1
    d. 0

26. Consider the vector space C over R and let $T: C \to C$ be a linear transformation given by $T(z) = \bar{z}$. Then which one of the following is correct?
    a. T is one-one, but not onto
    b. T is onto, but not one-one
    c. T is one-one as well as onto
    d. T is neither one-one nor onto.

27. If T is a linear transformation from a real vector space R^2 to a real vector space R^3 such that $T(x, y) = (x - y, y - x, -x)$, then what is the nullity of T?
    a. 0
    b. 1
    c. 2
    d. 3

28. If n is a positive integer and $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then what is A^n equal to ?
    a. $\begin{bmatrix}\cos n\theta & -\sin n\theta\\ \sin n\theta & \cos n\theta\end{bmatrix}$
    b. $\begin{bmatrix}-\cos n\theta & \sin n\theta\\ \sin n\theta & \cos n\theta\end{bmatrix}$
    c. $\begin{bmatrix}\cos n\theta & \sin n\theta\\ \sin n\theta & -\cos n\theta\end{bmatrix}$
    d. $\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$

29. If A and B are symmetric matrices of the same order, then which one of the following is not correct ?
    a. $A + B$ is a symmetric matrix.
    b. $AB - BA$ is a symmetric matrix.
    c. $AB + BA$ is a symmetric matrix.
    d. $A + A^T$ and $B + B^T$ are symmetric matrices.

30. If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ satisfies the matrix equation $A^2 - KA + 2I = 0$, then what is the value of k?
    a. 0
    b. 1
    c. 2
    d. 3

31. What is the value of the determinant $\begin{vmatrix}1 & b+c & b^2+c^2\\ 1 & c+a & c^2+a^2\\ 1 & c+a & a^2+b^2\end{vmatrix}$?
    a. $(a - b)(b - c)(c - a)$
    b. $(a + b)(b + c)(c + a)$
    c. abc
    d. $a + b + c$

32. Under which one of the following conditions does the system of equations $\begin{bmatrix}1 & 2 & 4\\ 2 & 1 & 2\\ 1 & 2 & a-4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}6\\ 4\\ a\end{bmatrix}$ have a unique solution?
    a. For all $a \in R$
    b. $a = 8$
    c. For all $a \in Z$
    d. $a \neq 8$

33. Consider the equations $2x + 2y = 1$ and $2x - y = 1$ over Z_3. What is the solution of (x, y)?
    a. (1, 1) but not (2, 0)
    b. (2, 0) but not (1, 1)
    c. Both (1, 1) and (2, 0)
    d. (1/2, 0)

34. Which one of the following is correct? For different values of a and b, the straight line

given by $x(a + 2b)\ y(a - 3b) = a - b$ passes through

a. A conjugae point
b. A fixed point
c. The origin
d. None of these

35. The line $3x + 2y = 24$ meets the y-axis at A and the x-axis at B, and perpendicular bisector of AB meets the line through (0, -1) parallel to the x-axis at C. What is the area of the triangle ABC?

a. 91 sq unit
b. 81 sq unit
c. 61 sq unit
d. 41 sq unit

36. Consider the following statements

S_1 : The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight line.

S_2: The equation $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin.

Which one of the following is correct?

a. If S_1 is true, S_2 is always true.
b. If S_1 is not true, then S_2 is also not true.
c. S_2 is always true and S_1 implies S_2 if $c = 0$.
d. Both S_1 and S_2 imply each other.

37. What is the angle between the two tangents drawn from (1, 0) to the curve $y^2 - 4x = 0$?

a. 30°
b. 45°
c. 60°
d. 90°

38. If a circle and the rectangular hyperbola $xy = c^2$ meet in four points t_1, t_2, t_3 and t_4, then what is $t_1t_2t_3t_4$ equal to?

a. 1
b. –1
c. 2
d. –2

39. If P, Q, A, B are (1, 2, 5), (-2, 1, 3), (4, 4, 2), (2, 1, -4) respectively, then what is the projection of PQ on AB?

a. 3
b. 7/2
c. 4
d. 9/2

40. What is the equation of the plane which bisects the line joining the points (3, -2, 1) and (1, 4, -3) at right angles?

a. $x - 3y + 2z + 3 = 0$
b. $3x - 2y + z + 3 = 0$
c. $x + 4y - 3z + 2 = 0$
d. $x - 3y + 2z + 2 = 0$

41. What is the equation of the plane which passes through the z-axis and is perpendicular to the line $\frac{x-a}{\cos\theta} = \frac{y+2}{\sin\theta} = \frac{z-3}{0}$?

a. $x + y\tan\theta = 0$
b. $y + x\tan\theta = 0$
c. $x\cos\theta + y\sin\theta = 0$
d. $x\sin\theta - y\cos\theta = 0$

42. A straight line L in the XY-plane bisects the angle between OX and OY. What are the direction cosines of L?

a. $\langle 1/\sqrt{2}, 1/\sqrt{2}, 0\rangle$
b. $\langle 1/2, \sqrt{3}/2, 0\rangle$
c. $\langle 0, 0, 1\rangle$
d. $\langle 2/3, 2/3, 1/3\rangle$

43. What is the equation of the cone with vertex at origin and passing through the circle $x^2 + y^2 = 4, z = 2$?

a. $x^2 + y^2 + z^2 = 4$
b. $x^2 + y^2 - z^2 = 0$
c. $x^2 + y^2 - z^2 = 2$
d. $x^2 + y^2 + z^2 = 2$

44. If $\vec{a}, \vec{b}, \vec{c}$ are non-zero vectors such that $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$, then which one of the following is correct?

a. $\vec{a}$ and $\vec{b}$ are collinear.
b. $\vec{a}$ and $\vec{c}$ are collinear.
c. $\vec{b}$ and $\vec{c}$ are collinear.
d. None of these.

45. Consider the following statements

S_1: $\vec{a}, \vec{b}, \vec{c}$ are non-zero, non coplanar vectors.

S_2: $\vec{a} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ $\vec{b} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

$\vec{c'} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ are non coplanar.

Which one of the following is correct?

a. S_1 implies S_2 but S_2 does not imply S_1.
b. S_1 does not imply S_2 but S_2 implies S_1.
c. S_1 implies S_2 and S_2 implies S_1.
d. S_1 does not imply S_2 and S_2 does not imply S_1.

46. What is the volume of the tetrahedron with vertices at (0, 0, 0), (1, 1, 1), (2, 1, 1) and (1, 2, 1)?

a. 1/6
b. 1/3
c. ½
d. 1

47. If $\vec{r}$ satisfies the equation $\vec{r} \times (\hat{i} + 2\hat{j} + \hat{k}) = \hat{i} - \hat{k}$, then for any scalar m, what is $\vec{r}$ equal to?

a. $\hat{i} + m(\hat{i} + 2\hat{j} + \hat{k})$
b. $\hat{j} + m(\hat{i} + 2\hat{j} + \hat{k})$
c. $\hat{k} + m(\hat{i} + 2\hat{j} + \hat{k})$
d. $\hat{i} - \hat{k} + m(\hat{i} + 2\hat{j} + \hat{k})$

48. For the triangle OBC, one vertex O is the origin and the position vectors of the other vertices B and C are $\vec{b}$ and $\vec{c}$ respectively and a, b, c are the lengths of the sides BC, OB, and OC respectively. What is the position vector of the incentre of the triangle OBC?

a. $\frac{b\vec{b} + c\vec{c}}{b + c}$
b. $\frac{b\vec{b} + c\vec{c}}{a + b + c}$
c. $\frac{c\vec{b} + b\vec{c}}{a + b + c}$
d. $\frac{b\vec{b} + c\vec{c}}{a + b + c}$

49. What is the range of the function $f(x) = \log_2\{(\sin x - \cos x + 3\sqrt{2})/\sqrt{2}\}$?

a. [1, 2]
b. [0, 1]
c. (1, 2)
d. (0, 1)

50. If $\lim_{x \to 0} \frac{(x + 3\sin x - x^3 - k\sinh x)}{1 - \cos x + x^2 - 3x^3}$ exists then what is the value of k?

a. –1
b. 2
c. 3
d. 4

51. If $f(x) = \begin{cases} \{\sin(a+2)x + \sin x\}/x & x < 0 \\ b, & x = 0 \\ \{(x + 3x^2)^{1/3} - x^{1/3}\}/x^{\frac{4}{3}} & x > 0 \end{cases}$ is continuous at x = 0, then what are the values of a and b respectively?

a. –1, -1
b. 1, -1
c. 2, 1
d. -2, 1

52. If $f(x) = x^n|x|$ for x, f(x) is differentiable at the origin if n is equal to which one of the following?

a. –1
b. 0
c. any real number
d. any positive integer

53. What is the maximum value of $y = \sin^3 x \cos x, 0 < x < \pi$?

a. $-3\sqrt{3}/16$
b. $3\sqrt{3}/4$
c. $-3/16$
d. $3\sqrt{3}/16$

54. Match List – I with List-II and select the correct answer using the code given below the lists

List I

A. The function $x^3 - 6x^2 - 36x + 7$ increase when
B. The function $x^3 - 6x^2 - 36x + 7$ is maximum at
C. The function $x^3 - 6x^2 - 36x + 7$ is minimum at
D. The function $x^3 - 6x^2 - 36x + 7$

List II

1. x = -2
2. x = 6

3. $x < -2$ or $x > 6$
4. $-2 < x < 6$

Codes:

	A	B	C	D
b.	4	2	1	3
c.	3	1	2	4
d.	3	2	1	4
e.	4	1	2	3

55. If 4a + 2b + c = 0, then the equation 3ax2 + 2bx + c = 0 has at least one real root lying between which of the following?

a. 0 and 1
b. 1 and 2
c. 0 and 2
d. none of these.

56. Under which one of the following conditions does the function $f(x) = \{(x^2)^m \sin(x^{-2})n\}$, $x \neq 0, n > 0$ and f(0) = 0, have a derivative at x = 0?

a. $m \geq -1/2$
b. $m > 0$
c. $m > ½$
d. $m \geq 1/2$

57. If the length to the curve $f(x) = x^2$ any point {c, f(c)} is parallel to the joining the point {a, f(a)} and {b, f(b)} on the curve, then which one of the ollowing is correct?

a. a, c, b are in AP
b. a, c, b are in GP
c. a, c, b are in HP
d. a, c, b do not follow definite sequence.

58. What is the maximum area of the rectangle whose sides pass through the angular points of a given rectangle of sides 'a' and 'b'?

a. $(a + b)^2/2$
b. $(a + b)^2$
c. $(a - b)^2/2$
d. $(a^2 + b^2)$

59. What is the abscissa of the point at which the tangent to the curve $y = e^x$ is parallel to the chord joining the extremities of the curve in the interval [0, 1]?

a. ½
b. ln (1/e)
c. ln (e − 1)
d. 1/e

60. What is the subnormal at $x = \pi/2$ on the curve y = x sin x?

a. 1
b. $2/\pi$
c. $\pi/2$
d. 2

61. Which one of the following is correct? The inclined asymptotes of the curve $x^3 - xy^2 - 2xy + 2x - y = 0$ are themselves.

a. perpendicular
b. parallel
c. inclined at an angle $\pi/3$.
d. Inclined at an angle $\pi/4$

62. Which one of the following pertaining to the tangent at any point on an ecurve $x^{2/3} + y^{2/3} = a^{2/3}$ is correct?

a. sum of its intercepts made with the coordinate axes is constant.
b. It encloses a triangle of constant area with the coordinate axes.
c. Length of its portion intercepted between the coordinate axes is constant.
d. It always passes through the origin.

63. What is the least absolute value of the radius of explanation curvature for the curve y = ln x?

a. $3\sqrt{3}$
b. $2\sqrt{3}$
c. $\sqrt{3}/\sqrt{2}$
d. $3\sqrt{3}/2$

64. What is the value of $\int_0^{\pi/2} \frac{5\sin x + 3\cos x}{\sin x + \cos x} dx$?

a. 0
b. $\pi/2$
c. 4π
d. 2π

65. The maximum value of f(x), where $f(x) = \int_0^x \sin\{x(1-x)\}dx$ occurs at which one of the following points?

a. x = 0
b. x = 1
c. x = -1
d. none of these

66. What is the volume of solid generated, when the area of the ellipse $(x^2/9) + (y^2/4) = 1$ (in the first quadrant) is revolved about y-axis?

a. 16π

b. 12π
c. 8π
d. 6π

67. If $\int_0^1 x^m(1-x)^n dx = \int_0^1 x^n(1-x)^p dx$, then what is p equal to?
a. 2n
b. m
c. m + n
d. m/n

68. What is the area of the region bounded by the curve $2y = x - 3x - 2y^2$ and the x-axis?
a. 125/48 sq unit
b. 4 sq unit
c. 3 sq unit
d. 125/24 sq unit

69. What is the value of $\lim_{x\to y}\frac{x^x - y^x}{x^y - y^y}$?
a. $\frac{1+\ln y}{1-\ln y}$
b. $\frac{1-\ln y}{1+\ln y}$
c. $\frac{-1+\ln y}{1+\ln y}$
d. $\frac{-1-\ln y}{1-\ln y}$

70. If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is a constant, then what is the value of $\frac{d^3}{dx^3}\{f(x)\}$ at x = 0?
a. p
b. $p + p^3$
c. $p - p^3$
d. Independent of p

71. What are the order and degree respectively of the differential equation of the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is an arbitrary constant?
a. 1, 1
b. 1, 2
c. 1, 3
d. 2, 1

72. What is the solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$, with the given conditions y(0) = 0 and y'(0) = 1?
a. $y = e^{-x}\cos x$
b. $y = e^{-x}\sin x$
c. $y = (\cos x + \sin x)e^{-x}$
d. $y = \sin x$

73. What is the solution of the differential equation $(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$?
a. $x + ye^{x/y} = c$
b. $y + xe^{x/y} = c$
c. $x - ye^{x/y} = c$
d. none of these

74. The singular solution of the differential equation $y = px + f(p)$ will be obtained by eliminating p between the equation $y = px + f(p)$ and which one of the following equations?
a. $x + \frac{df}{dp} = 0$
b. $\frac{dy}{dp} = x + \frac{df}{dp}$
c. $\frac{dy}{dx} = p$
d. $\frac{dy}{dx} = p + \frac{df}{dp}$

75. Consider the following statements in respect of the differential equation $2xy\frac{dy}{dx} = y^2 - x^2$ is;
i. The differential equation is a homogeneous equation
ii. The curve represented by the differential equation is a family of circles.
iii. The differential equation of its orthogonal trajectories is $\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$

Which one of the following is correct?
a. i and ii only
b. i and iii only
c. ii and iii only
d. i, ii and iii

76. What are the orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2 = \frac{a}{x}$?

a. $9a(y + c)^2 = \pm 2x^{3/2}$

b. $9a(y + c)^2 = \pm 2x^{2/3}$

c. $9a(y + c)^3 = 4x^2$

d. $9a(y + c)^2 = 4x^2$

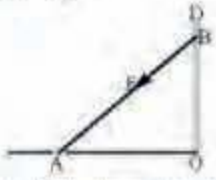
77. From a square lamina abcd shose diagonals meet at O, the triangle AOB is cut and the remaining part is hung up at D. In the position of equilibrium, how much angle does DC make with the vertical?

a. $\tan^{-1}(7/9)$

b. $\tan^{-1}(5/9)$

c. 45°

d. 30°



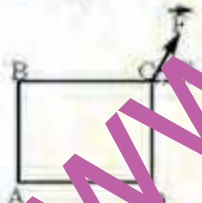
78. A pillar OD is to be pulled down by tying a rope of length l = AB to some point B of the pillar and then pulling the rope with a force F as shown in the above figure. F will have maximum moment about O when OB equals to which one of the following ?

a. $\sqrt{2}l$

b. $l/\sqrt{2}$

c. $\sqrt{3}l$

d. $l/\sqrt{3}$



79. A force $\vec{F}$, having magnitude of 10 dyne, is applied on the corner C of a rectangular plate ABCD, as shown in the above figure. If AB = 8 cm, AD = 12 cm, then what is the moment of $\vec{F}$ about A?

a. $20(-2+3\sqrt{3}) \times 10^{-7}$ Nm

b. $20(-2+3\sqrt{3}) \times 10^{-5}$ Nm

c. $20(2+3\sqrt{3}) \times 10^{-7}$ Nm

d. $20(2+3\sqrt{3}) \times 10^{-5}$ Nm

80. A heavy spherical ball of weight W is on a smooth inclined plane (α = angle of inclination of the plane to the horizontal). A force of magnitude P is applied through the centre of the ball in order to maintain the ball at rest. What is the value of P?

a. $P = W\sqrt{1+\cos^2\alpha}$

b. $P = W\cos\alpha$

c. $P = W\sin\alpha$

d. $P = W\sqrt{1+\sin^2\alpha}$

81. The weight of a triangular lamina ABC is 9 g. What is the additional weight to be placed at A so that the new centre of gravity divides the median through A in the ratio 3 : 4?

a. 2g

b. 3g

c. 4g

d. 5g

82. Two spheres of radii 6 cm, 3 cm are firmly united. The two spheres are solid and of the same material. What is the distance of the centre of gravity of the whole body from the centre of the larger sphere?

a. 1 cm

b. 2 cm

c. 3 cm

d. 4 cm

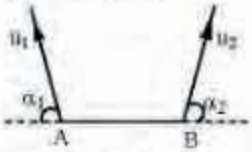
83. If the angle of friction is λ, then what is the greatest height at which a particle can rest inside a hollow sphere of radius a?

a. $a\sin\lambda$

b. $a(1-\cos\lambda)$

c. $a\tan\lambda$

d. $a(1-\sin\lambda)$



84. Two points A and B have velocities u_1 and u_2 as shown in the figure above. If AB = d, what is the angular velocity of A relative to B?

a. $(u_1\cos\alpha_1 - u_2\cos\alpha_2)/d$

b. $(u_1\cos\alpha_1 + u_2\cos\alpha_2)/d$

c. $(u_1\sin\alpha_1 - u_2\sin\alpha_2)/d$

d. $(u_1\sin\alpha_1 + u_2\sin\alpha_2)/d$

85. Two particles are projected vertically upwards from a place at an interval of 2 seconds. If the first and the second particle attain the respective greatest heights H_1 and H_2 simultaneously, then which one of the following is correct ?

a. $\sqrt{H_2} = (\sqrt{H_1} + \sqrt{2g})$

b. $\sqrt{H_1} = (\sqrt{H_2} + \sqrt{2g})$

c. $\sqrt{H_1 / H_2} = 2g$

d. $\sqrt{H_1 / H_2} = 2$

86. A particle of unit mass is constrained to move in a smooth circular path of radius α with constant speed. If now an additional radial force of magnitude P acts on the particle, how does the kinetic energy (E) of the particle change?

a. E changes by $P\alpha/2$

b. E changes by $\sqrt{2P\alpha}$

c. E changes by $P\alpha/4$

d. E changes by $2P\alpha$

87. A smooth heavy bead moves along a wire, which is bent in a circle of radius α in a vertical plane. The bead starts from the rest position where the radius to it makes an angle of 60° with the vertical. What is the velocity of the bead when it reaches the lowest point (the wire is fixed in space)?

a. $\sqrt{3g\alpha}$

b. $\sqrt{2g\alpha}$

c. $2\sqrt{g\alpha}$

d. $\sqrt{5g\alpha}$

88. A particle is projected with velocity v at an angle (< 45°) to the horizontal and reaches a point on the horizontal distant R from the point of projection. What is the greatest height (h) attained during the path of the projectile ?

a. $h = \frac{v^2}{4g}\left(1 - \sqrt{1 - \left(\frac{g^2R^2}{v^4}\right)}\right)$

b. $h = \frac{v^2}{4g}\left(1 + \sqrt{1 - \left(\frac{g^2R^2}{v^4}\right)}\right)$

c. $h = \frac{v^2}{2g}\left(1 - \sqrt{1 - \left(\frac{g^2R^2}{v^4}\right)}\right)$

d. $h = \frac{v^2}{2g}\left(1 + \sqrt{1 - \left(\frac{g^2R^2}{v^4}\right)}\right)$

89. A particle is executing simple harmonic motion and its displacement from its mean position is given by $x = \alpha\cos(nt + k)$, where t denotes the time and a, n, k are positive constants. Under what condition will the speed of the particle be maximum?

a. $t = (2p+1)\pi / 2n$, p being an integer

b. $t = (2p+1)\pi / 2n - (k/n)$, p being an integer

c. $t = (2p+1)\pi / 2n + (k/n)$, p being an integer

d. $t = p\pi / n - (k/n)$, p being an integer

90. A particle whose weight on the surface of the earth is W falls to the surface of the earth from a height equal to the diameter 2R of the earth. What is the work done by the earth's attraction?

a. 2RW

b. 2RW/3

c. 4RW/3

d. 3RW/2

91. A floppy with 1.44 MB capacity can store the information equivalent to which one of the following?

a. 1.44×2^6 bytes

b. 1.44×2^{10} bytes

c. 1.44×2^{20} bytes

d. 1.44×10^{24} bytes

92. Under what conditions of the inputs a and B, will the output in the gates for operations OR and XOR be different?

a. A = 1, B = 0

b. A = 0, B = 1

c. A = 0, B = 0

d. A = 1, B = 1

93. **Step 1:** get A, B

Comment : A(i, j) and B(i, j) are m × n and n × p matrices.

Step 2: 1 to m

Do for j = 1 to p

Do C(i, j) ← 0

For k = 1 to n

Do C(i, j) ← X

Step 3: Output C

Comment : C=C(i, j) is the product matrix AB of the order m × p.

What is X in the above algorithm?

a. C(i, j)+ A(i, k)*B(k, j)

b. C(i, j)+ A(i, k)*B(j, k)

c. A(i, k)*B(k, j)

d. C(i, j)+ A(i, j)*B(i, j)

94. What is the decimal equivalent of the hexadecimal number FF ?

a. 225

b. 245

c. 255

d. 256

95. Which one of the called "coincidence detector" ?

a. OR gate

b. NAND gate

c. NOT gate

d. AND gate

96. Let $n \geq 3$, n be odd.

Assertion (A): For any $i = 1, 2, \ldots, n-1$, if $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the roots of the equation $x^n - x - 1 = 0$ then $(1+\alpha_1)(1+\alpha_2)\ldots(1+\alpha_n) = 1$

Reason (R) : If $\alpha_1, \ldots, \alpha_n$ are the roots of the equations, $x^n - x - 1 = 0$, and then $(1+\alpha_1)(1+\alpha_2)\ldots(1+\alpha_n) = 1$

a. Both A and R are individually true and R is the correct explanation of A

b. Both A and R are individually true but R is the correct explanation of A

c. A is true but R is false

d. A is false but R is true.

97. **Assertion (A):** There is at least one cyclic group of order 100 which has only 5 subgroups.

Reason (R) : A finite cyclic group of order m has a unique subgroup of order n, where n is a divisor of m.

a. Both A and R are individually true and R is the correct explanation of A

b. Both A and R are individually true but R is the correct explanation of A

c. A is true but R is false

d. A is false but R is true.

98. **Assertion (A):** The function $f(x) = \frac{x}{1+|x|}$ is not differentiable at x = 0.

Reason (R) : $|x|$ and hence $(1+|x|)$ is not differentiable at x = 0.

a. Both A and R are individually true and R is the correct explanation of A

b. Both A and R are individually true but R is the correct explanation of A

c. A is true but R is false

d. A is false but R is true.

99. **Assertion (A):** The function $y = x^2/4$ is a singular solution of $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

Reason (R) : The general solution of the given equation is $y = cx - c^2$ and the given solution cannot be obtained by assigning a definite value to c in the general solution.

a. Both A and R are individually true and R is the correct explanation of A

b. Both A and R are individually true but R is the correct explanation of A

c. A is true but R is false

d. A is false but R is true.

100. **Assertion (A):** $\int_0^{\pi} \cos^4 x\, dx = 2\int_0^{\pi/2} \cos^4 x\, dx$

Reason (R) : The integrand is an even function.

a. Both A and R are individually true and R is the correct explanation of A

b. Both A and R are individually true but R is the correct explanation of A

c. A is true but R is false

d. A is false but R is true.