PRACTICE PAPER 2

SECTION-I

Straight Objective Type

This section contains 9 multiple choice question numbered 1 to 9. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Q1

Let a, b, c be three real numbers such that a< b< c. Let f(x) be continuous in [a, c] and differentiable is (a, c). If f'(x) is strictly increasing in (a, c), then

a. $(c-b)f(a)+(b-a)\,f(c)>(c-a)\,f(b)$

b. $(c-b)f(a)+(b-a)\,f\,(c)<(c-a)\,f(b)$

c. $f(a)<\,f(b)<\,f(c)$

d. None of the above

Q2

The number of rational points on the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is

a. none

b. two

c. four

d. Infinite

Q3

The number of non-negative continuous functions on [0, 1] satisfying $\int_0^1 f(x)\,dx = 1, \int_0^1 xf(x)\,dx = a \; and \int_0^1 x^2\,f(x)\,dx = a^2, (a \neq 0)$ is

a. none

b. two

c. four

d. Infinite

Q4

The hypotenuse of a right angled triangle passes through the point (2, 4) and the sides are along x and y-axis. The number of such triangles having area 18 units must be

a. 1

b. 2

c. 3

d. 4

Q5

Three numbers are drawn from the set $\{1, 2, 3, \dots, n\}$ with replacement. The probability that their sum is 2n, is

a. $\frac{1}{2_n}$

b. $\frac{1}{2}$

c. $\frac{(n-1)(n+4)}{2n^3}$

d. $\frac{(n-1)(n+2)}{n^3}$

Q6

Among the following points there is only point from which tangents can be drawn to the ellipse $3x^2 + 4y^2 = 12$ are perpendicular. The point must be

a. $(3, 4)$

b. $(3, \sqrt{2})$

c. $(-2, \sqrt{3})$

d. $(1, 2)$

Q7

The minimum value of $\sin 3A + \sin 3B + \sin 3C$ in a triangle must be

a. -1

b. -2

c. $-\frac{\sqrt{3}}{2}$

d. None of the above

Q8

If the perimeter of a triangle is 2 then the expression $E = ab + bc + ac - abc - 1$

a. is essentially positive

b. is essentially negative

c. may or may not be positive

d. None of these

Q9

Let $S = \sum_{r=1}^{2000} \sqrt{1 + \frac{1}{r^2} + \frac{1}{(r+1)^2}}$. Then S must be equal to

a. $2000\left(1 + \frac{1}{2001}\right)$

b. $2000\left(1 + \frac{1}{1999}\right)$

c. $\sqrt{2000}\left(1 + \frac{1}{2001}\right)$

d. None of these

Section-II

Multiple Objective Type

Q10

The values of a for which $x^4 - 2ax^2 + x + a^2 - a = 0$ has all real roots are

a. -1

b. 1

c. 2

d. 3

Q11

If y is a function of x given by $2 \log (y-1) - \log x - log(y-2) = 0$, then

a. domain is[4, ∞]

b. domain is [0, ∞]

c. range is (2, ∞)

d. range is (0, ∞)

Q12

If ordered pair (∝, β) where ∝, β ∈ 1 satisfy the equation $2x^2 - 3xy - 2y^2 = 7$, then value of ∝ + β can be

a. 5

b. 4

c. −4

d. 3

Q13

For the parabola $y^2 = 4x$, let P be the point of concurrency of three normals and S be the focus. If $\propto_1$ be the sum of the angles made by three normals from the positive direction of x-axis and $\propto_2$ be the angle made by PS with the positive direction of x-axis then can be equal to

a. 1

b. 2

c. 1/2

d. 3/2

Q14

Let $\left(\frac{p_1}{q_1}, \frac{p_2}{q_2}\right)$ and $\left(\frac{a_1}{b_1}, \frac{a_2}{b_2}\right)$ be any two rational points on the circle $x^2 + y^2 = 1$ where $p_1, p_2, q_1, q_2, a_1, a_2, b_1$ and b_2 are integers and H.C.F. of (p_1, q_1), (p_2, q_2), (a_1, b_1) and (a_2, b_2) is 1. Then the statements which are always correct are

a. $q_1 = q_2$

b. $p_1 = \pm 1 \; or \; 0$

c. $b_1 = b_2$

d. $a_1 = \pm 1 \; or \; 0$

Q15

If all the roots of the equation $(x^2 - mx + n)(x^2 - nx + m) = 0$ are positive integers then m + n can be equal to

a. 8

b. 9

c. 10

d. 11

Q16

Let a, b, c be the lengths of the sides of a triangle *ABC* such that $b + c \neq 1, c - b \neq 1$. If $log_{b\,+\,c}\, a + log_{c-b}\, a = 2log_{c+b}alog_{c-b}a$, then

a. $sin^2A + sin^2B = sin^2C$

b. $\tan A + \tan B = 1$

c. $A + B = C$

d. $cos^2A + cos^2B = 1$

Q17

If $f(x) = \int_{x"}^{x"} \frac{dt}{In\, t}, x > 0\ \ and\, n > m$, then

a. $f'(x) = \frac{x^{m-1}(x-1)}{In\, x}$

b. $f(x)$ is decreasing for$x > 1$

c. $f(x)$is increasing in (0, 1)

d. $f(x)$is increasing for $x > 1$

Section-III

Assertion-Reason Type

Q18

Statement-1:

If n is odd then the product$P = (1 - i_1)(2 - i_2)(3 - i_3) \dots (n - i_n)$ where $i_1, i_2, i_3, \dots, i_n$are distinct integers taken from the set {1, 2, 3, …, n} is certainly even. Because

Statement-2:

P can be zero for some choice of $i_1, i_2, \dots, i_n$.

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

c. Statement-1 is True, Statement-2 is False

d. Statement-1 is True, Statement-2 is True

Q19

Statement-1:

For any large positive integer n, the integer next to $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{2n-1}-\frac{1}{2n}$ is 2.

Statement-2:

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots-\frac{1}{2n}=\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n-1}$$

Q20

Statement-1:

If n leaves remainder 2 when divided by 3 then 3^n-1 leaves remainder 8 when divided by 13. because

Statement-2:

$3^5-1=242$ leaves remainder 8 when divided by 13.

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1 c. Statement-1 is True, Statement-2 is False
d. Statement-1 is True, Statement-2 is True

Q21

Statement-2:

$\tan^{-1}\frac{1}{5}$Is approximately equal to$\frac{\pi}{16}$. because

Statement-1:

If$5+i=\sqrt{26}(\cos\theta+i\sin\theta)$, then$(5+i)^4=476+480\,i=676\,(\cos 4\theta+i\sin 4\theta)$

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
c. Statement-1 is True, Statement-2 is False
d. Statement-1 is True, Statement-2 is True

Section-IV

Linked Comprehension Type

$M_{22\text{-}24}$: Paragraph for Question Nos. 22 to 24

Let $I_{m,\ n} = \int_0^{\pi/2} cos^m\, x \cos nx\, dx$

Q22

If m and n are non-negative integers, then

a. $I_{m,\ n} > 0\ for\ all\ m\ and\ n$

b. $I_{m,\ n} = 0\ for\ some\ m\ and\ n$

c. $I_{m,\ n} < 0\ for\ some\ m\ and\ n$

d. None of these

Q23

$\frac{I_{m,\ n}}{I_{m-2,\ n}}$ must be equal to

a. $\frac{m(m-1)}{m^2+n^2}$

b. $\frac{m(m-1)}{m^2-n^2}$

c. $\frac{m(m-2)}{m^2-n^2}$

d. None of these

Q24

$I_{n,\ n}$ must be equal to

a. $\frac{\pi}{4}$

b. $\frac{\pi}{2n}$

c. $\frac{\pi}{2^{n+1}}$

d. None of these

$M_{25\text{-}27}$: Paragraph for Question Nos. 25 to 27

Let ∝, β, y be positive roots of the equation $x^3 + ax^2 + bx + c = 0$. answer the following questions

Q25

If $c = -1/64$ then minimum value of $\propto + \beta + y$ must be

a. 1/3

b. ¼

c. ½

d. ¾

Q26

If a = −1 then maximum value of $\propto \beta^2 y^3$ must be

a. 3/2

b. 1/2

c. 1/432

d. 1/64

Q27

If $c = -1/64$ such that $(\propto +\beta)^3 - 27 \propto \beta\, \gamma \leq 0$ then (a + b) must be equal to

a. $-\frac{9}{16}$

b. $\frac{9}{16}$

c. $-\frac{9}{32}$

d. $-\frac{3}{4}$

Section-V

Subject Type

This section contains 3 questions. Write the answer of the questions (28-31) from the following combinations:

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Q28

If a, b, c are positive integers which are in increasing G.P. If $log_6\, a + log_6\, b + log_6\, c = 6\ and\ b - a$ is a perfect cube then the numerical value of a + b + c must be equal to

Q29

If $x, y, z > 0$ then minimum value of $\frac{x^4+y^4+z^4}{xyz}\ is\ \sqrt{\lambda}$ the λ must be equal to

Q30

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \sin 60° & \cos 60° \\ -\cos 60° & \sin 60° \end{bmatrix}$. Then $(BB^TA)^{2007} = \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix}$, the numerical quantity λ must be

Q31

If the roots of the equation $x^2 + ax + b + 1 = 0$ are distinct positive integers, then min. value $a^2 + b^2$ must be equal to

PRACTICE PAPER 2-SOLUTIONS

ANSWER KEY

SECTION I

1.(a)

2.(d)

3.(b)

4.(b)

5.(c)

6.(c)

7.(d)

8.(a)

9.(a)

SECTION II

10.(b), (c)

11.(a), (c)

12.(b), (c)

13.(a), (b)

14.(a), (c)

15.(a), (d)

16.(a), (d)

17.(c), (d)

SECTION III

18.(b)

19.(a)

20.(b)

21.(a)

SECTION IV

22.(c)

23.(b)

24.(c)

25.(d)

26.(c)

27.(a)

SECTION V

28. (0189)

29. (0008)

30.(2007)

31.(0010)