Introduction

The branches of electricity and magnetism were unified by scientists like Oersted, Rowland, Faraday, Maxwell and Lorentz.

The branch of physics covering a combined study of electricity and magnetism is known as electromagnetism or electrodynamics. It is useful in study of subjects like plasma physics, magneto-hydrodynamics and communication.

5.1 Oersted's Observation

In 1819 A.D., Oersted, a school teacher from Denmark, observed that magnetic field is produced around a wire carrying electric current. If a conducting wire is kept parallel to the magnetic needle and electric current is passed through it, needle gets deflected and aligns itself perpendicularly to the length of the wire.

5.2 Biot-Savart's Law

The intensity of magnetic field due to a current element $I\vec{dl}$ at a point having position vector $\vec{r}$ with respect to the electric current element is given by the formula

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3} \quad \text{and} \quad |\vec{dB}| = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2},$$

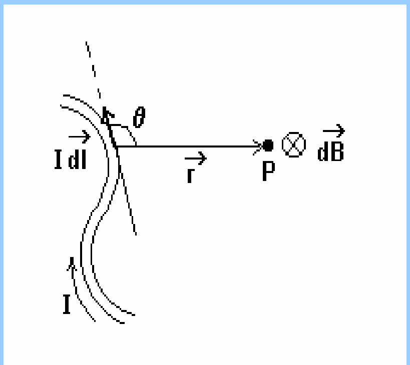
where $\vec{dB}$ = magnetic intensity in tesla (T) or weber / m^2,

$I\vec{dl}$ = current element (product of electric current and length of small line element $\vec{dl}$ of the conductor)

μ_0 = magnetic permeability of vacuum = $4\pi\times10^{-7}$ tesla metre per ampere ($T\,m\,A^{-1}$)

$\hat{r}$ = unit vector along the direction of $\vec{r}$ = $\dfrac{\vec{r}}{|\vec{r}|}$

and θ = angle between $\vec{dl}$ and $\vec{r}$



The direction of $\vec{dB}$ is perpendicular to the plane formed by $\vec{dl}$ and $\vec{r}$. As $\vec{dl}$ and $\vec{r}$ are taken in the plane of the figure, the direction of $\vec{dB}$ is perpendicular to the plane of the figure and going inside it, as shown by ⊗.

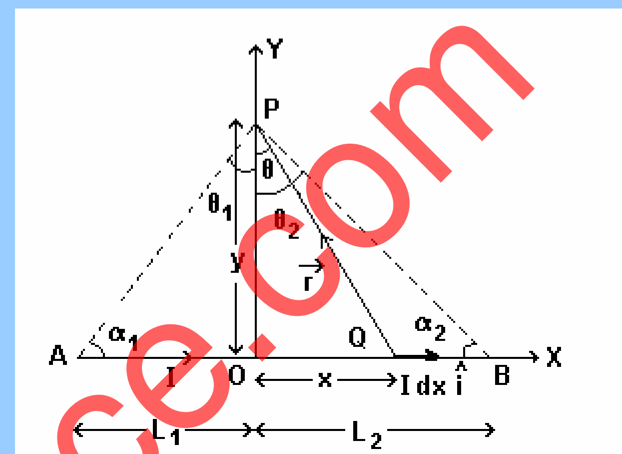
On integrating the above equation, we get the total intensity at the point P due to the entire length of the conducting wire as

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{\vec{dl}\times\hat{r}}{r^2} \quad \text{or} \quad \vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{\vec{dl}\times\vec{r}}{r^3}$$

5.3 Some Applications of Biot-Savart's Law

5.3 (a) Magnetic field due to a straight conductor carrying electric current

A straight conductor AB carrying electric current I is kept along X-axis as shown in the figure. It is desired to find magnetic intensity at a point P located at a perpendicular distance y from the wire. Y-axis is along OP.



A small current element $I\,dx\,\hat{i}$ is at a distance x from the origin on the wire.

By Biot-Savart's law, magnetic intensity at point P due to this current element is

$$\vec{dB} = \frac{\mu_0}{4\pi}\,\frac{I\,\vec{dx}\times\vec{r}}{r^3} \quad \dots \dots \ (1)$$

Putting $\vec{dx} = dx\,\hat{i}$ and $\vec{r} = \left(y\,\hat{j} - x\,\hat{i} \right)$

(from Δ OPQ formed by vectors)

$$\vec{dx} \times \vec{r} = dx\,\hat{i} \times \left(y\,\hat{j} - x\,\hat{i} \right) = y\,dx\,\hat{k}$$

$$\therefore \quad \vec{dB} = \frac{\mu_0}{4\pi}\,\frac{I\,y\,dx\,\hat{k}}{r^3}$$

This field is perpendicular to the plane formed by $\vec{dx}$ and $\vec{r}$ coming out normally from the plane of the figure. Integrating over the whole length of the wire,

$$\vec{B} = \int \vec{dB} = \frac{\mu_0 I y}{4\pi}\left[\int \frac{dx}{r^3}\right]\hat{k}$$

From the geometry of the figure, $r^2 = x^2 + y^2$ $\quad \therefore \ r^3 = (x^2 + y^2)^{3/2}$

$$\therefore \quad \vec{B} = \frac{\mu_0 I y}{4\pi}\left[\int \frac{dx}{(x^2 + y^2)^{\frac{3}{2}}}\right]\hat{k}$$

Putting $x = y\tan\theta$, $dx = y\sec^2\theta\,d\theta$ and integrating over the whole length of the wire, i.e., from $\theta = -\theta_1$ to $\theta = \theta_2$,

$$\vec{B} = \frac{\mu_0 I y}{4\pi}\left[\int_{-\theta_1}^{\theta_2} \frac{y\sec^2\theta\,d\theta}{\left[y^2\tan^2\theta + y^2\right]^{\frac{3}{2}}}\right]\hat{k} = \frac{\mu_0 I}{4\pi y}\left[\int_{-\theta_1}^{\theta_2} \cos\theta\,d\theta\right]\hat{k}$$

$$\therefore \quad \vec{B} = \frac{\mu_0 I}{4 \pi y} \left[\sin\theta \right]_{-\theta_1}^{\theta_2} \hat{k} = \frac{\mu_0 I}{4 \pi y} \left[\sin\theta_2 + \sin\theta_1 \right] \hat{k}$$

If the angles subtended by P at the ends A and B of the wire are α_1 and α_2, then

$$\vec{B} = \frac{\mu_0 I}{4 \pi y} \left[\cos\alpha_1 + \cos\alpha_2 \right] \hat{k} = \frac{\mu_0 I}{4 \pi y} \left[\frac{L_1}{\sqrt{y^2 + L_1^2}} + \frac{L_2}{\sqrt{y^2 + L_2^2}} \right] \hat{k}$$

If O is the midpoint of the wire, i.e., if OP is the perpendicular bisector, $L_1 = L_2 = L/2$,

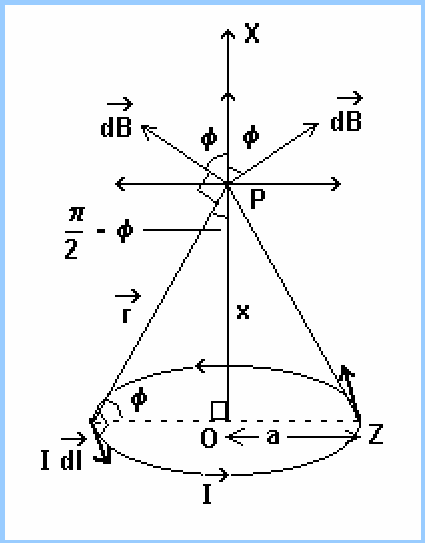
$$\therefore \quad \vec{B} = \frac{\mu_0 I}{4 \pi y} \frac{2L}{\sqrt{4y^2 + L^2}} \hat{k}$$

Putting $\theta_1 = \theta_2 = \pi/2$ for an infinitely long wire,

$$\vec{B} = \frac{\mu_0 I}{2 \pi y} \hat{k}$$

To decide the direction of the magnetic field, right hand thumb rule can be used. If the wire is held in right hand such that the thumb is in the direction of the electric current, the fingers encircling the wire indicate the direction of the magnetic field lines.

5.3 (b) Magnetic Field at Any Point on the Axis of a Circular Ring carrying Current:



Consider a point P on the axis of a ring carrying current I at a distance x from its centre and having position vector $\vec{r}$ with respect to an element of length $\vec{dl}$ of the ring.

The magnetic field $\vec{dB}$ at the point P, due to the current element $I\vec{dl}$, is in a direction perpendicular to the plane formed by $\vec{dl}$ and $\vec{r}$. It can be resolved into two mutually perpendicular components:

(i) dB cos ϕ parallel to X-axis and
(ii) dB sin ϕ perpendicular to X-axis.

All dB sin ϕ components due to the diametrically opposite elements nullify each other, whereas the axial components dB cos ϕ add up.

$$\therefore \quad dB(x) = \left|\vec{dB}\right| \cos\phi = \left| \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \vec{r}}{r^3} \right| \cos\phi = \frac{\mu_0}{4\pi} \frac{I\, dl \,.\, r \sin\theta}{r^3} \cos\phi$$

But $\vec{dl} \perp \vec{r}$ $\quad \therefore \sin\theta = \sin\frac{\pi}{2} = 1$ and $\cos\phi = \frac{a}{r}$ (from the figure)

$$\therefore \quad dB(x) = \frac{\mu_0 I}{4\pi r^2}\frac{a}{r}\,dl$$

Integrating over the circumference of the ring, the resultant magnetic field at point P is

$$B(x) = \frac{\mu_0 Ia}{4\pi r^3}\oint_{ring} dl = \frac{\mu_0 Ia}{4\pi r^3}\,2\pi a$$

From geometry of the figure, $r^2 = a^2 + x^2 \Rightarrow r^3 = (a^2 + x^2)^{3/2}$

$$\therefore \quad B(x) = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

The direction of the magnetic field is along X-axis given by the right hand thumb rule. On curling the fingers of right hand in the direction of flow of electric current, the thumb stretching perpendicularly to the plane of the circle formed by the fingers indicate the direction of the magnetic field.

If the ring consists of N closely wound turns,

$$B(x) = \frac{\mu_0 N I a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

Taking x = 0, magnetic intensity at the centre of the ring is

$$B(\text{centre}) = \frac{\mu_0 N I}{2a}$$

For a point far away from the centre of the coil as compared to its radius, $x >> a$. Neglecting a^2 in comparison to x^2, magnetic intensity at a distance x from the centre on the axis of the coil is

$$B(x) = \frac{\mu_0 N I a^2}{2x^3}$$

To find the direction of the magnetic field on the axis of the ring, curl the fingers of right hand in the direction of flow of electric current. The thumb stretching perpendicularly to the plane of the circle formed by the fingers indicates the direction of the magnetic field.

5.3 (c) Solenoid:

A helical coil consisting of closely wound turns of insulated conducting wire is called a solenoid.

Fig. 1 represents a cross section along the length of the solenoid. The $\times$ signs indicate the wires going into the paper and the $\bullet$ signs indicate wires coming out of the plane of paper. The axis of the solenoid coincides with X-axis and radius of the solenoid is a. P is a point inside the solenoid on its axis as shown in Fig. 2.

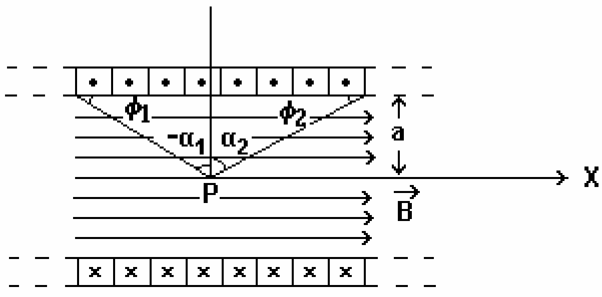


Fig. 1

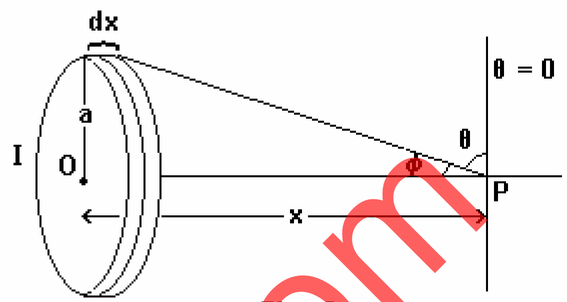


Fig. 2

To find magnetic intensity at P, consider a small part of a solenoid of width dx, at a distance x from point P. It can be regarded as a thin ring. If there are n turns per unit length of the solenoid, there will be ndx number of turns in this part. Hence magnetic intensity at P due to this ring will be

$$dB(x) = \frac{\mu_0 I a^2 n dx}{2(a^2 + x^2)^{\frac{3}{2}}}$$

From Fig. 2, $x = a \tan\theta$, $\therefore$ $dx = a \sec^2\theta \, d\theta$ and $a^2 + x^2 = a^2 + a^2 \tan^2\theta = a^2 \sec^2\theta$

Integrating over the entire length of the solenoid, i.e., from $\theta = -\alpha_1$ to $\theta = \alpha_2$, magnetic intensity at point P is

$$B = \int_{-\alpha_1}^{\alpha_2} \frac{\mu_0 I a^2 n (a \sec^2\theta) d\theta}{2 a^3 \sec^3\theta} = \frac{\mu_0 n I}{2} \int_{-\alpha_1}^{\alpha_2} \cos\theta \, d\theta$$

$$= \frac{\mu_0 n I}{2} [\sin\theta]_{-\alpha_1}^{\alpha_2} = \frac{\mu_0 n I}{2} (\sin\alpha_2 + \sin\alpha_1)$$

In terms of angles ϕ_1 and ϕ_2 as shown in Fig. 1,

$$B = \frac{\mu_0 n I}{2} (\cos\phi_2 + \cos\phi_1)$$

For a very long solenoid (in principle of infinite length), $\alpha_1 = \alpha_2 = \pi / 2$

$\therefore$ $B = \mu_0 n I$, where n = number of turns per unit length of the solenoid.

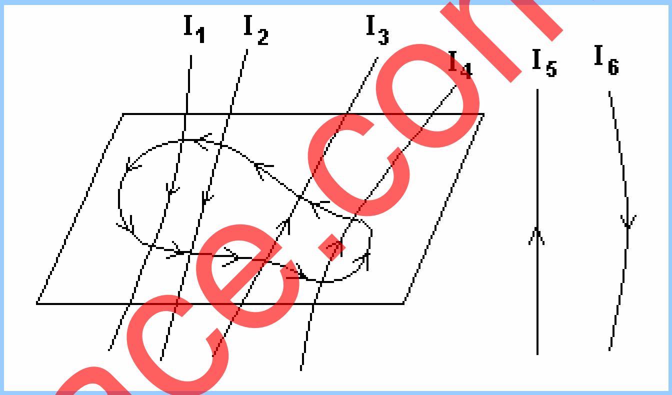
For a very long solenoid, magnetic field is uniform inside the solenoid and zero outside the solenoid just as for a capacitor of very large plates electric field is uniform in the inner region and zero outside the plates.

Thus a capacitor can be used where a uniform electric field is required and a solenoid is used where uniform magnetic field is required.

5.4 Ampere's Circuital Law

The statement of Ampere's circuital law is:

" The line integral of magnetic induction over a closed loop in a magnetic field is equal to the product of algebraic sum of electric currents enclosed by the loop and the magnetic permeability."



Mathematically, $\oint \vec{B} \cdot \vec{dl} = \mu_0 \sum I$

To decide the sign convention for electric currents, consider the magnetic field loop produced by electric currents as shown in the figure.

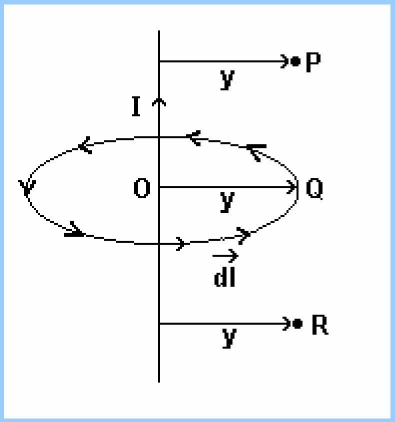
Arrange a right-handed screw perpendicular to the plane containing closed magnetic loop and rotate it in the direction of vector line elements taken for line integration. Electric currents in the direction of advancement of the screw are considered positive and the currents in the opposite direction are considered negative.

Hence, algebraic sum of currents enclosed by the closed magnetic intensity loop is

$$\sum I = I_3 + I_4 - I_1 - I_2$$

Here, currents outside the loop are not to be considered even though they contribute in producing the magnetic field.

<u>(1) To find magnetic field due to a very long and straight conductor carrying electric current, using Ampere's law:</u>



Consider a very long (in principle infinitely long) straight conductor carrying electric current I **as shown in the figure.**

As the wire is infinitely long, points P, Q and R which are at the same perpendicular distance y from it will have the same magnetic intensity. In fact, all points on the loop of radius y passing through point Q will have the same magnetic field. If this magnetic field is $\vec{B}$**, then applying Ampere's law to the loop,**

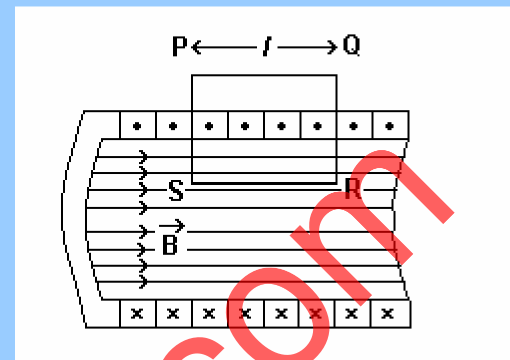
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \sum I \quad \Rightarrow \quad \oint B \, dl \cos\theta = \mu_0 I$$

As $\vec{B}$ **and** $\vec{dl}$ **are in the same direction at every element,** $\cos\theta = \cos 0 = 1$**.**

$$\therefore \oint B \, dl = \mu_0 I \qquad \therefore B \oint dl = \mu_0 I \quad (\text{as } B \text{ is constant})$$

$$\therefore B \cdot 2\pi y = \mu_0 I \qquad \therefore B = \frac{\mu_0 I}{2\pi y}$$

(2) Formula of a solenoid:



The figure shows the cross-section of a very long solenoid. It is desired to find the magnetic intensity at point S lying inside the solenoid.

Taking line integral over Ampearean loop PQRS shown in the figure,

$$\oint \vec{B}\cdot\vec{dl} = \int_P^S \vec{B}\cdot\vec{dl} + \int_S^R \vec{B}\cdot\vec{dl} + \int_R^Q \vec{B}\cdot\vec{dl} + \int_Q^P \vec{B}\cdot\vec{dl}$$

The magnetic field on part PQ of the loop will be zero as it is lying outside the solenoid. Also some part of QR and SP is outside and the part inside is perpendicular to the magnetic field. Hence magnetic field on them is zero.

$$\therefore \quad \int_Q^P \vec{B}\cdot\vec{dl} = \int_R^Q \vec{B}\cdot\vec{dl} = \int_P^S \vec{B}\cdot\vec{dl} = 0$$

$$\therefore \quad \oint \vec{B}\cdot\vec{dl} = \int_S^R B\,dl\cos 0^\circ = B\int_S^R dl = B\,l$$

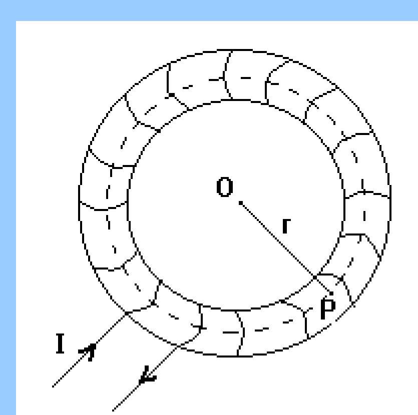
If n = number of turns per unit length of the solenoid, then the number of turns passing through the Ampearean loop is $n\,l$. Current passing through each turn is I, so total current passing through the loop is $n\,l\,I$.

From Ampere's circuital law, $\oint \vec{B}\cdot\vec{dl} = \mu_0 n\,l\,I$

$$\therefore \quad B\,l = \mu_0 n\,l\,I \quad \text{and} \quad B = \mu_0 n\,I$$

This method is valid only for very long solenoid in which all points inside the solenoid can be considered equivalent and is not advisable to use for a solenoid of finite length.

5.5 Toroid



If the solenoid is bent in the form of a circle and its two ends are connected to each other then the device is called a toroid. It can be prepared by closely winding an insulated conducting wire around a non-conducting hollow ring.

The magnetic field produced inside the toroid carrying electric current can be obtained using Ampere's circuital law.

To find a magnetic field at a point P inside a toroid which is at a distance r from its centre, consider a circle of radius r with its centre at O as an Amperean loop. By symmetry, the magnitude of the magnetic field at every point on the loop is the same and is directed towards the tangent to the circle.

$$\therefore \quad \oint \vec{B}\cdot\vec{dl} = \oint B\,dl = B\oint dl = B(2\pi r) \quad \dots \quad \dots \quad \dots \quad (1)$$

If the total number of turns is N and current is I, the total current through the said loop is NI. From Ampere's circuital law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 N I \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Comparing equations (1) and (2), $B(2\pi r) = \mu_0 N I$

$\therefore \quad B = \dfrac{\mu_0 N I}{2\pi r} = \mu_0 n I$, ($n = \dfrac{N}{2\pi r}$ is the number of turns per unit length of the toroid)

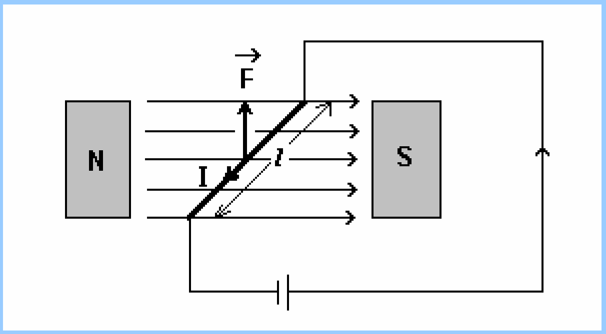
In an ideal toroid, where the turns are completely circular, magnetic field at the centre and outside the toroid is zero. In practice, the coil is helical and hence a small magnetic field exists outside the toroid.

Toroid is a very important component of Tokamak used for research in nuclear fusion.

5.6 Force on a current carrying wire placed in a magnetic field

Ampere showed that "two parallel wires placed near each other exert an attractive force if they are carrying currents in the same direction, and a repulsive force if they are carrying currents in the opposite directions."

A magnetic field is created around the wire carrying an electric current. If another wire carrying some current is placed in its neighbourhood, then it experiences a force. The law giving this force was given by Ampere as under.



The force acting on a current element $I\,\vec{dl}$ due to the magnetic induction $\vec{B}$ is

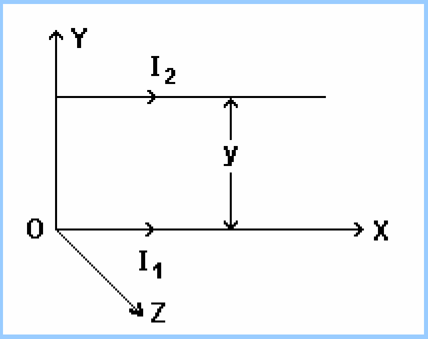
$$\vec{dF} = I\,\vec{dl} \times \vec{B}$$

If a straight wire of length l carrying a current I is placed in a uniform magnetic field $\vec{B}$, the force acting on the wire can be given by

$\vec{F} = I\,\vec{l} \times \vec{B}$ Such an arrangement is shown in the figure. The direction of force can be determined using the right hand screw rule.

5.6 (a) The formula for the force between two conducting wires placed parallel to each other and carrying currents in the same direction:

Consider two very long conducting wires placed parallel to each other along X-axis, separated by a distance y and carrying the currents I_1 and I_2 in the same direction as shown in the figure.



Magnetic field at a distance y from the conductor carrying current I_1 is

$$\vec{B} = \frac{\mu_0 I_1}{2\pi y} \hat{k}$$

The strength of this field is the same at all the points of the wire carrying the current I_1 and is directed along Z-axis. Therefore, the force acting on the second wire over its length l will be

$$\vec{F} = I_2 \vec{l} \times \vec{B} = I_1 I_2 \frac{\mu_0}{2\pi y} \vec{l} \times \hat{k} = I_1 I_2 \frac{\mu_0}{2\pi y} l \hat{i} \times \hat{k}$$

$$\therefore \vec{F} = \frac{\mu_0}{2\pi} \frac{I_1 I_2 l}{y} \hat{j}$$

The force $\vec{F}$ acts along negative y-direction which indicates that it is attractive.

5.6 (b) Definition of ampere

In the equation, $\vec{F} = \frac{\mu_0}{2\pi} \frac{I_1 I_2 l}{y} \hat{j}$, if $I_1 = I_2 = 1$ A, $y = 1$ m and $l = 1$ m, then

$$|\vec{F}| = \frac{\mu_0}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ N}$$

Based on this, SI definition of 1 ampere curent is given as under:

When the magnetic force acting per meter length in two infinitely long wires of negligible cross-sectional area placed parallel to each other at a distance of 1 meter in vacuum carrying identical currents is 2×10^{-7} N, the current passing through each wire is 1 ampere.

5.7 Force on an electric charge moving in a magnetic field: Lorentz Force

The current I flowing through a conductor of cross-sectional area A is given by

$I = n A v_d q$, where q = charge on the positively charged particle,
n = number of free charge carriers per unit volume of the conductor,
v_d = drift velocity.

$$\therefore I \vec{dl} = q n A v_d \vec{dl} = q n A \vec{v_d} dl \quad (\because v_d \text{ and } dl \text{ are in the same direction})$$

When this conductor is placed in a magnetic field of intensity $\vec{B}$, the force acting on it is

$$\vec{dF} = I \vec{dl} \times \vec{B} = q n A dl (\vec{v_d} \times \vec{B})$$

But n A dl = total number of charged particles on the current element

$\therefore$ the magnetic force acting on a charged particle of charge q is given by

$$\vec{F_m} = \frac{\vec{dF}}{nAdl} = \frac{q n A dl (\vec{v_d} \times \vec{B})}{n A dl} = q (\vec{v_d} \times \vec{B})$$

The magnetic force acting on a charge moving through a magnetic field is perpendicular to the velocity of the particle. Work done by this force is zero and hence the kinetic energy of the particle remains constant. Only the direction of velocity goes on changing at every instant.

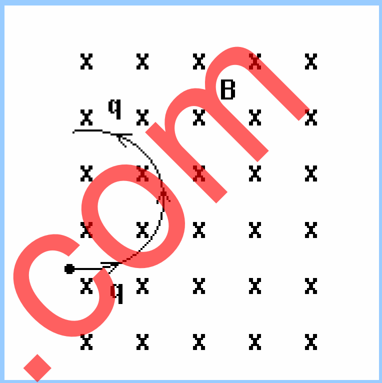
If an electric field $\vec{E}$ is also present alongwith $\vec{B}$, the resultant force acting on the charged particle will be

$\vec{F} = \vec{F_e} + \vec{F_m} = q [\vec{E} + \vec{v_d} \times \vec{B}]$. This force is known as Lorentz Force.

5.8 Cyclotron

Scientists E. O. Lawrence and M. S. Livingston constructed the first cyclotron in 1934 A. D. which is used to accelerate charged particles.

To understand how a cyclotron works, consider the motion of a positively charged particle moving with velocity $\vec{v}$ and entering perpendicularly uniform magnetic field of intensity $\vec{B}$ as shown in the figure.



The force acting on the charged particle is

$$\vec{F} = q(\vec{v} \times \vec{B}) = q\,v\,B \sin\theta = q\,v\,B \quad (\because \sin\theta = \pi/2)$$

Under the effect of this force, the charged particle performs uniform circular motion in a plane perpendicular to the plane formed by v and B.

$$\therefore \quad q\,v\,B = \frac{m\,v^2}{r} \quad \text{and} \quad r = \frac{m\,v}{q\,B} = \frac{p}{q\,B},$$

where p is the linear momentum of the charged particle.

Putting $v = r\,\omega_c$, where ω_c is called the angular frequency of the cyclotron,

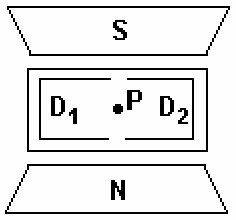
$$r = \frac{m\,r\,\omega_c}{q\,B} \quad \therefore \quad \omega_c = \frac{q\,B}{m} \quad \text{and} \quad f_c = \frac{q\,B}{2\,\pi\,m}$$

The equation shows that the frequency does not depend on the momentum. Hence on increasing momentum of the particle, the radius of its circular path increases but its frequency does not. This fact is used in the design of the cyclotron.

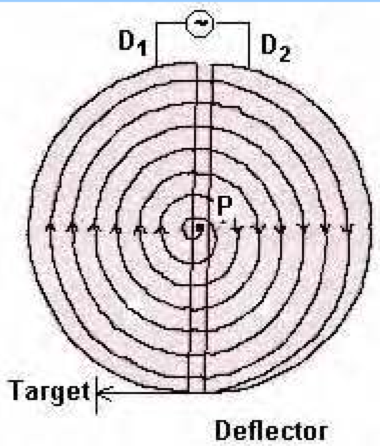
The figure shows side view and top view of a cyclotron.

Construction:

Two D-shaped boxes are kept with their diameters facing each other with a small gap as shown in the figure. A uniform magnetic field is developed in the space enveloped by the two boxes with a strong electromagnet. These two boxes are called Dees as they are D-shaped.



Side View of Cyclotron



Top View of Cyclotron

An A.C. of high frequency is applied between the two Dees. The device is kept in an evacuated chamber in order to avoid the collision of charged particles with the air molecules.

Working:

A positively charged particle is released at the centre P of the gap at time t = 0. It gets attracted towards the Dee which is at a negative potential at that time. It enters the uniform magnetic field between the Dees perpendicularly and performs uniform circular motion in the gap. As there is no electric field inside the Dees, it moves on a circular path of radius depending upon its momentum and comes out of the Dee after completing a half circle.

As the frequency of A.C. (f_A) is equal to f_c, the diameter of the opposite Dee becomes negative when the particle emerges from one Dee and attracts it with a force which increases its momentum. The particle then enters the other Dee with larger velocity and hence moves on a circular path of larger radius. This process keeps on repeating and the particle gains momentum and hence radius of its circular path goes on increasing but the frequency remains the same. Thus the charged particle goes on gaining energy which becomes maximum on reaching the circumference of the Dee.

When the particle is at the edge, it is deflected with the help of another magnetic field, brought out and allowed to hit the target.

Such accelerated particles are used in the study of nuclear reactions, preparation of artificial radioactive substances, treatment of cancer and ion implantation in solids.

Limitations:

- According to the theory of relativity, as velocity of the particle approaches that of light, its mass goes on increasing. In this situation, the condition of resonance ($f_A = f_c$) is not satisfied.
- To accelerate very light particles like electrons, A.C. of very high frequency (of the order of GHz) is required.
- It is difficult to maintain a uniform magnetic field over large sized Dees. Hence accelerators like synchroton are developed.

5.9 Torque acting on a rectangular coil, carrying electric current and suspended in a uniform magnetic field:

One turn of rectangular coil has length, QR = l and width, PQ = b (see Fig. 1)

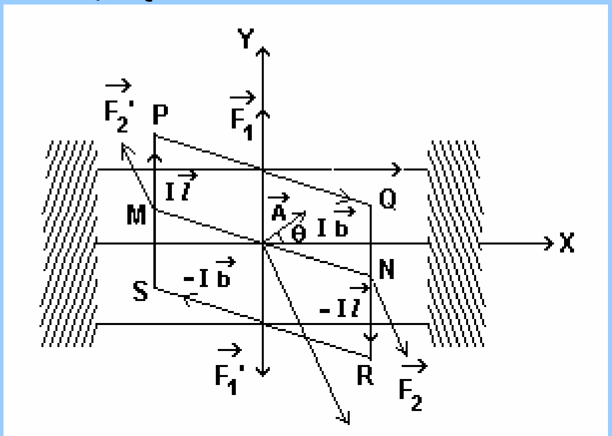


Fig. 1

Magnetic field taken along X-axis is $\vec{B} = B\hat{i}$

Force acting on side PQ forming current element $I\vec{b}$ is $\vec{F_1} = I\vec{b} \times \vec{B}$

Similarly, force acting on side RS forming current element $-I\vec{b}$ is $\vec{F_1'} = -I\vec{b} \times \vec{B}$

The forces $\vec{F_1}$ and $\vec{F_1'}$ are equal in magnitude, opposite in direction and collinear. Hence, they cancel each other.

Now, force acting on side QR forming current element $-I\,l\,\hat{j}$ is $\vec{F_2} = -I\,l\,\hat{j} \times B\,\hat{i} = I\,l\,B\,\hat{k}$

Similarly, force acting on side SP forming current element $I\,l\,\hat{j}$ is $\vec{F_2}' = I\,l\,\hat{j} \times B\,\hat{i}$

$= -I\,l\,B\,\hat{k}$

The forces $\vec{F_2}$ and $\vec{F_2}'$ are equal in magnitude, opposite in direction but are non-collinear. So they give rise to a torque (couple).

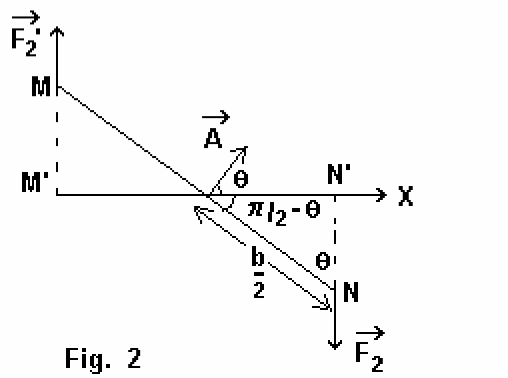


Fig. 2

Fig. 2 shows the top view of the coil. Here $\vec{A}$ is the area vector of the coil which makes an angle θ with the magnetic intensity $\vec{B}$ along X-axis.

By definition, magnitude of torque (couple)

= magnitude of a force × perpendicular distance between the two forces

$\therefore\ |\vec{\tau}| = |\vec{F_2}|\ M'N'$

$= I\,l\,b\,B \sin\theta$

$\therefore\ |\vec{\tau}| = N\,I\,l\,b\,B \sin\theta$ (for a coil having N turns)

Expressing area A of the coil in vector form

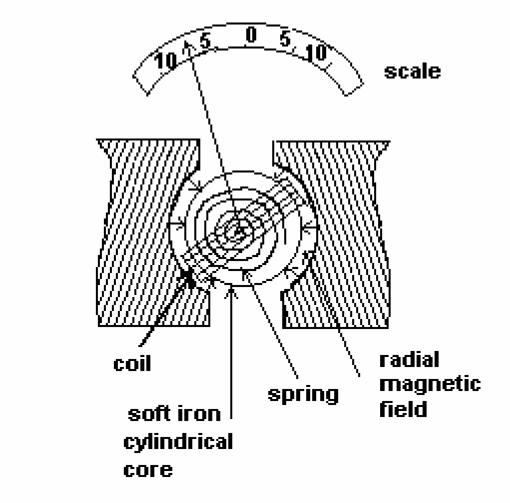
$\vec{\tau} = N\,I\,\vec{A} \times \vec{B}$

$= \vec{\mu} \times \vec{B}$, where $\vec{\mu} = N\,I\,\vec{A}$ is called the “magnetic moment” linked with the coil.

This equation is valid for a coil of any other shape. The direction of torque is along Y-axis. The direction of magnetic moment, $\vec{\mu}$, can be determined using the right hand screw rule.

5.10 Galvanometer

A galvanometer is a device used to detect current. With appropriate modification, it can be converted into an ammeter which can measure currents of the order of an ampere or milliammeter to measure currents in the range of milliamperes or microammeter to measure microampere currents.



Construction:

A light rectangular frame on which a coil of thin copper wire is wound is pivoted between two almost frictionless pivots and placed between cylindrical poles of a permanent magnet, so that it can freely rotate in the region between the poles. The poles are suitably shaped and a small soft iron cylindrical core is placed at the axis of the coil (free from the coil) to obtain uniform magnetic field.

When the current is passed through the coil, a torque acts on it and is deflected. This deflection causes the restoring torque in the spiral springs attached at the two ends of the coil and the coil attains a steady deflection. The pointer attached to the coil moves on a scale and indicates the current.

Principle and Working

The torque developed in the coil due to the current passing through it is given by

$\tau = N I A B \sin\theta$, where N = number of turns in the coil,
I = current through the coil,
A = area of the coil,
B = magnetic intensity of the field and
θ = angle between area vector of the coil and the direction of magnetic intensity

As the magnetic field is radial, angle between $\vec{A}$ and $\vec{B}$ is 90° in any position of the coil and sin 90° being 1,

$\tau = N I A B$

The restoring torque produced in the springs is directly proportional to the deflection ϕ of the coil.

$\therefore \tau$ (restoring) $= k\phi$, where k = effective torsional constant of the springs.

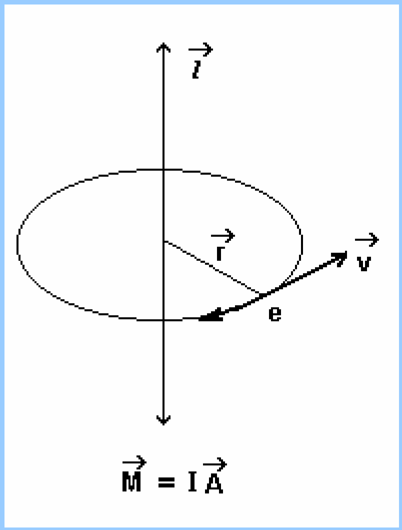
For steady deflection ϕ, $N I A B = k\phi$

$\therefore I = \left[\dfrac{k}{NAB}\right]\phi$ or, $I \propto \phi$

The scale of the galvanometer can be appropriately calibrated to measure the current.

To measure very weak currents of the order of 10^{-11} A, the galvanometers with coils suspended by an elastic fibre between appropriately designed magnetic poles are used.

5.11 Orbital magnetic moment of an electron revolving in an orbit of an atom



The magnetic dipole moment of the loop of area A carrying current I is given by

$M = IA$ (per turn)

If an electron is revolving in an orbit of radius r with frequency f, it will pass through a point of its orbit f times in one second. In this case, the charge passing through that point in one second is ef where e = charge of an electron. This constitutes an electric current

$I = ef$. Taking $A = \pi r^2$,

$M = ef(\pi r^2)$

$$\therefore M = \frac{e\omega\pi r^2}{2\pi} = \frac{e\omega r^2}{2} = \frac{em\omega r^2}{2m} = \frac{e}{2m}l$$

where $l = m \omega r^2$ = angular momentum of electron and
m = mass of an electron.

Expressing in vector form,

$$\vec{M} = -\left(\frac{e}{2m}\right)\vec{l}$$

Here $\vec{M}$ and $\vec{l}$ are in mutually opposite directions according to the right hand screw rule. Hence, negative sign appears in the above equation.

The ratio $\frac{e}{2m}$ is a constant called the gyro-magnetic ratio and its value is

8.8×10^{10} C kg^{-1}.