

PHYSICS

Paper I

Time Allowed : Three Hours

Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt Questions No. 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

All questions carry equal marks.

The number of marks carried by each part of a question is indicated against each.

Answers must be written in ENGLISH only.

Assume suitable data, if necessary, and indicate the same clearly.

Unless otherwise indicated, symbolic notations carry usual meaning.

Useful Constants :

$$\text{Electron charge (e)} = 1.602 \times 10^{-19} \text{ C}$$

$$\text{Electron rest mass (m}_e\text{)} = 9.109 \times 10^{-31} \text{ kg}$$

$$\text{Proton mass (m}_p\text{)} = 1.672 \times 10^{-27} \text{ kg}$$

$$\text{Vacuum permittivity (}\epsilon_0\text{)} = 8.854 \times 10^{-12} \text{ farad/m}$$

$$\text{Vacuum permeability (}\mu_0\text{)} = 1.257 \times 10^{-6} \text{ henry/m}$$

Velocity of light in free space (c) = 3×10^8 m/s

Boltzmann constant (k) = 1.38×10^{-23} J/K

Electron volt (eV) = 1.602×10^{-19} J

Planck's constant (h) = 6.62×10^{-34} J-s

Stefan's constant (σ) = 5.67×10^{-8} W m⁻² K⁻⁴

Avogadro's number (N) = 6.02×10^{26} kmol⁻¹

Gas constant (R) = 8.31×10^3 J kmol⁻¹ K⁻¹

exp (1) = 2.7183

SECTION A

1. Answer any *four* of the following :

4×10=40

- (a) A particle of mass 'm' moves according to the equations $x = a \cos \omega t$, $y = a \sin \omega t$, $z = c$, where a , c , ω are constants. Obtain the instantaneous velocity and linear momentum vectors in terms of the Cartesian components and hence the angular momentum. Find the force \vec{F} and the torque \vec{N} acting on the particle and verify that the angular momentum \vec{L} and the torque \vec{N} satisfy the relation $\frac{d\vec{L}}{dt} = \vec{N}$.

- (b) A particle of rest mass 'M' moving at a velocity 'u' collides with a stationary particle of rest mass 'm'. If the particles stick together, show that the speed of the composite ball is equal to

$$v = \frac{u \gamma M}{(\gamma M + m)}, \text{ where } \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

- (c) A plano-convex lens of crown glass is connected to a concavo-convex lens of flint glass so that the convex surface of the first and the concave surface of the second fit exactly. If the combination is achromatic and the combined focal length is -50 cm, determine the radii of curvature of the faces of the flint glass lens. The dispersive power of the crown glass is one-half of that of the flint glass and the refractive indices for yellow light are respectively 1.500 and 1.625 respectively.
- (d) A diffraction grating has 5000 lines per cm. For illumination at normal incidence, determine the dispersive power of the grating in the second order spectrum in the range of wavelengths around 500 nm.

- (e) Consider a Gaussian pulse propagating in pure silica, at $\lambda_0 = 0.85 \mu\text{m}$, along z-direction. As the pulse propagates, it gets broadened according to the formula

$$\tau^2(z) = \tau_0 \left(1 + \frac{4\alpha^2 z^2}{\tau_0^4} \right), \quad \alpha = \left. \frac{d^2 k}{d\omega^2} \right|_{\omega=\omega_0}$$

where $\tau(z)$ is the pulse width after propagation through a distance z and τ_0 is the pulse width at $z = 0$. Given that $\alpha = 3 \times 10^{-26} \frac{\text{S}^2}{\text{m}}$ at $\lambda_0 = 0.85 \mu\text{m}$. Calculate the distance at which $\tau(z) = \sqrt{2} \tau_0$.

2. (a) A rigid body rotates with the angular velocity ' ω ' about an axis through the origin O and having direction cosines l, m, n . Show that the moment of inertia of the rigid body about the axis is

$$I = I_{xx}l^2 + I_{yy}m^2 + I_{zz}n^2 + 2I_{xy}lm + 2I_{yz}mn + 2I_{zx}nl$$

where the symbols have their usual meanings. 20

- (b) The principal moments of inertia of a body at a point are given as 200, 300 and 450 gm.cm². Write down the equation of the ellipsoid of inertia at that point. 10

- (c) A light spring of relaxed length ' a_0 ' is suspended from a point. It carries a mass ' m ' at its lower free end, which stretches it through a distance l . Show that the vertical oscillation of the system is simple harmonic in nature and has time period $T = 2\pi\sqrt{l/g}$, where g is the acceleration due to gravity. 10

3. (a) In the steady state forced vibration of the damped harmonic oscillator, show that the amplitude of the driven system is maximum when $p = \sqrt{\omega^2 - 2b^2}$ and further the value of the maximum amplitude is $\frac{f}{2b\sqrt{\omega^2 - b^2}}$, where the symbols have their usual significance.
(Here you start from the steady state solution) 5
- (b) In a double slit experiment the slits are 2 mm apart and are illuminated with a mixture of two wavelengths, $\lambda = 750$ nm and $\lambda' = 900$ nm. At what minimum distance from the common central bright fringe on a screen 2 m distant from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other ? 10
- (c) What is Fermat's principle ? Derive the laws of reflection and refraction with the help of this principle, when the light is incident on a curved surface. 25
4. (a) Explain the diffraction at straight edge with the help of Cornu's spiral. 25
- (b) Distinguish between the intensity patterns due to diffraction from a narrow slit and a straight edge. 15

SECTION B

5. Answer any *four* questions from the following : $4 \times 10 = 40$

(a) The electric field in a medium is given by

$$\vec{E} = \vec{E}_0 e^{-\alpha z} \sin(kz - \omega t),$$

where \vec{E}_0 is a constant vector with dimensions of the electric field. Prove that \vec{E} cannot have a component along the unit vector, \hat{z} , parallel to the z-axis. Here α is a positive constant.

- (b) Show that the elemental quantity of heat δQ is not a total differential.
- (c) Prove that the Maxwell's equations in a medium contain the conservation of charge in differential form.
- (d) An inductor of inductance 5 H is suddenly connected to a 10 V d.c. power supply through a resistor of 10 Ω . After what time will the current in the circuit be 1/10th of its steady state value ?
- (e) Consider the Earth as a black body. Radiations from the Sun arrive at the surface of the Earth with an average intensity of S watts/m². If the reflection coefficient of the Earth's surface is α , determine the temperature of the Earth under equilibrium conditions.

6. (a) Write down the expression for the Bose – Einstein distribution function and explain the meaning of the symbols used. 10
- (b) Consider a gas of photons in equilibrium and contained in a volume V at a temperature T . Using the Bose – Einstein statistics, calculate the total energy of the photon gas. 20
- (c) If the magnetic field \vec{B} , at a point with position vector \vec{r} is uniform, show that the corresponding vector potential $\vec{A}(\vec{r})$ is given by

$$\vec{A}(\vec{r}) = -\frac{1}{2} \left[\vec{r} \times \vec{B} \right]. \quad 10$$

7. (a) Consider an infinite current sheet with a uniform current density \vec{K} (Amp/m). Show that the magnetic field \vec{H} at a point away from the sheet is

$$\vec{H} = \frac{1}{2} \vec{K} \times \hat{n}$$

where \hat{n} is a unit normal vector directed from the current sheet to the point. 25

- (b) The plane $y = 5$ carries a current of density $10 \hat{z}$ (Amp/m). Calculate the value of the magnetic field \vec{H} at the point $(0, 1, -5)$. 15

8. (a) Consider an infinite line charge with charge density ρ coulomb/meter located at a distance d meters from a grounded conducting plane $z = 0$. Determine :
- (i) the magnitude of the potential V for $z > 0$ and $z \leq 0$. 15
 - (ii) the surface charge density induced on the conducting plane. 10
- (b) The current density in spherical co-ordinates is given by

$$\vec{J} = \frac{1}{r^3} \left[2 \cos \theta \hat{r} + \sin \theta \hat{\theta} \right] \text{ A/m}^2$$

where \hat{r} and $\hat{\theta}$ are unit vectors. Calculate the amount of current passing through a hemisphere of radius 20 cm. 15

Sl. No.

7841

D-VSF-L-CUB

PHYSICS

Paper II

Time Allowed : Three Hours

Maximum Marks : 200

INSTRUCTIONS

*Candidates should attempt questions 1 and 5 which are compulsory, and any **THREE** of the remaining questions, selecting at least **ONE** question from each Section.*

All questions carry equal marks.

Marks allotted to parts of a question are indicated against each.

*Answers must be written in **ENGLISH** only.*

Assume suitable data, if considered necessary, and indicate the same clearly.

Neat sketches may be drawn, wherever required.

(Contd.)

List of Useful Constants :

Mass of proton	= 1.673×10^{-27} kg
Mass of neutron	= 1.675×10^{-27} kg
Mass of electron	= 9.11×10^{-31} kg
Planck constant	= 6.626×10^{-34} Js
Boltzmann constant	= 1.380×10^{-23} JK ⁻¹
Bohr magneton	= 9.273×10^{-24} A/m ²
Nuclear magneton (μ_N)	= 5.051×10^{-27} JT ⁻¹ (Nm ²)
Electronic charge	= 1.602×10^{-19} C
Atomic mass unit (u)	= 1.660×10^{-27} kg = 931 MeV

$$g_s^p = 5.5855 \mu_N$$

$$m(p) = 1.00727 \text{ (u)}$$

$$m(n) = 1.00866 \text{ (u)}$$

$$m\left({}_2^4\text{He}\right) = 4.002603 \text{ u}$$

$$m\left({}_6^{12}\text{C}\right) = 12.00000 \text{ u}$$

$$m\left({}_{38}^{87}\text{Sr}\right) = 86.908893 \text{ u}$$

Section 'A'

1. Answer any *four* of the following :

(a) On the basis of uncertainty principle calculate the size of Hydrogen atom. 10

(b) What are Pauli spin matrices ?

Show that :

$$\left(\vec{\sigma} \cdot \vec{A}\right)\left(\vec{\sigma} \cdot \vec{B}\right) = \vec{A} \cdot \vec{B} + i\vec{\sigma} \cdot \left(\vec{A} \times \vec{B}\right)$$

where $\vec{\sigma}$ are the Pauli spin matrices and \vec{A} and \vec{B} are vector operators which commute with $\vec{\sigma}$, but do not necessarily commute with each other. 10

(c) Obtain the term symbols for two singlet states and two triplet states for two electron atoms. 10

(d) The wavelengths (λ) in the visible spectrum of H atom can be expressed by the empirical formula

$$\lambda = \left(\frac{n_1^2}{n_1^2 - 4}\right) \cdot G$$

where n_1 is an integer and $n_1 = 3, 4$ etc. and G is an empirical constant.

Prove from the above, the wave number

$$\bar{\nu} = R_H \left(\frac{1}{2^2} - \frac{1}{n_1^2}\right), \text{ where } R_H = \frac{4}{G} \quad 10$$

- (e) What is the mechanism of emission of light in fluorescent lamps and in painted signboards? Explain. 10

2. (a) For a quantum mechanical system prove that all energy eigen-values E_n are real and if $E_n \neq E_k$, then the corresponding eigen functions are orthogonal. 10

- (b) Solve the Schrodinger equation for a potential step function given by

$$v(x) = 0 \text{ for } x < 0 \\ = v_0 \text{ for } x > 0$$

and calculate the reflection and transmission coefficients. Show that for $E < v_0$ there is a finite probability of finding the particle in a classically forbidden region. 20+10

3. (a) Solve the eigen value equation

$$L^2 Y(\theta, \phi) = \lambda \hbar^2 Y(\theta, \phi) \text{ and obtain the eigen values and eigen functions of } L^2. \quad 20$$

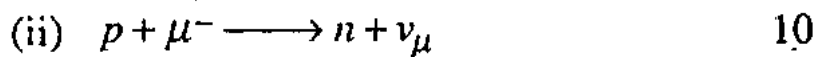
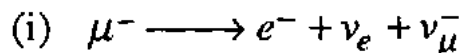
- (b) (i) What angles do the \vec{L} (vector) make with the z-axis when $l = 2$ for an electron?
- (ii) Determine the values of the total angular momentum for a 3d electron. 20

4. (a) Discuss the vibrational spectra of a diatomic molecule treating it as an anharmonic oscillator. 20
- (b) Give the elementary theory of NMR. Explain the two different relaxation processes. 20

Section 'B'

5. Answer any *four* of the following :

- (a) Find the density of $^{12}_6\text{C}$ nucleus. (Given, nuclear radius of carbon = 2.7×10^{-15} m) 10
- (b) Explain the source of energy in stars using proton-proton thermonuclear reaction cycles. 10
- (c) What type of interaction takes place in the following reactions and justify your answer.



- (d) How are the band structures of insulators and semiconductors, similar? How are they different? 10
- (e) (i) What are intrinsic and extrinsic semiconductors? Explain.
- (ii) How are n -type and p -type semiconductors obtained? 10
6. (a) What are magic numbers in an atomic nucleus and why are they so called? How is it explained using nuclear shell model? 20
- (b) What are the reactor materials used in design and construction of a nuclear reactor and their important functions? 20
7. (a) What are quarks? Give any four properties. Give quark composition of the following particles:
- $p, n, \Lambda^0, \pi^+, K^+$ 20
- (b) Using band theory of solids, explain whether the effective mass of an electron can be positive, negative as well as infinity. Explain the significance of negative mass. 20

8. (a) Using OP-AMPS design an analog circuit to solve the following differential equation :

$$\frac{d^2v}{dt^2} + k_1 \frac{dv}{dt} + k_2v = v_1(t)$$

with $\frac{dv(0)}{dt} = 1.5 \text{ V}; v(0) = 2.0 \text{ V}.$ 20

- (b) Explain the basic structure and operation of an *n*-channel FET.

How FET acts as an amplifier ? Explain. 20

