

(1) Find the area of the region bounded by the circle $x^2 + y^2 = r^2$.

[Ans: πr^2]

(2) Find the area of the region bounded by $y = x^2 - 5x + 4$ and X-axis.

[Ans: $\frac{9}{2}$]

(3) Find the area of the region enclosed by $y^2 = 8x$ and $x + y = 0$.

[Ans: $\frac{32}{3}$]

(4) Find the area of the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 4x$.

[Ans: $\frac{8\pi}{3} - 2\sqrt{3}$]

(5) Prove that the area of the region bounded by $y = 4x - x^2$ and X-axis is $\frac{32}{3}$.

(6) Find the volume of the solid obtained by revolution of portion of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ on right hand semi plane of Y-axis about Y-axis.

[Ans: $\frac{4}{3} \pi a^2 b$]

(7) If the region bounded by $y^2 = 8x$ from its vertex to $x = 2$ is rotated about X-axis, find the volume of the solid generated.

[Ans: 16π]

(8) Prove that the volume of the solid generated by revolving the region bounded by $y = x^2 + 1$ and $y = 2x + 1$ about X-axis is $\frac{104\pi}{15}$.

- (9) Find the volume of the right circular cone having semi-vertical angle α and radius of base equal to r .

$$\left[\text{Ans: } \frac{1}{3} \pi r^3 \cot \alpha \right]$$

- (10) Line $x = c$ divides the area of the region bounded by $y^2 = 4x$ and $x = 16$ in two regions having equal areas. Find c .

$$\left[\text{Ans: } 2 \frac{10}{3} \right]$$

- (11) Find the area of the region bounded by $y = x^2$ and the line $y = x + 2$.

$$\left[\text{Ans: } \frac{9}{2} \right]$$

- (12) Find the area of the region bounded by $y = 5x^2$ and $2x^2 - y + 9 = 0$.

$$[\text{Ans: } 12 \sqrt{3}]$$

- (13) The region bounded by $y = 2x^2$, X-axis and $x = 5$ is rotated about Y-axis. Find the volume of the solid generated.

$$[\text{Ans: } 625 \pi]$$

- (14) Find the volume of the solid generated when the region bounded by $y = x^2$ and $y = 4x - x^2$ is rotated about X-axis.

$$\left[\text{Ans: } \frac{32 \pi}{3} \right]$$

- (15) If the region bounded by $x^2 - y^2 = a^2$, $x = a$ and $x = 2a$ is rotated about Y-axis, find the volume of the solid of revolution.

$$[\text{Ans: } 4 \sqrt{3} \pi a^3]$$

(16) Prove that the area of the region enclosed by the circle $x^2 + y^2 = 64$ and parabola $y^2 = 12x$ is $\frac{16}{3}(4\pi + \sqrt{3})$.

(17) Prove that the area of the region bounded by $x = 6 + 4y - y^2$ and \overleftrightarrow{AB} where A is (4, 3) and B is (-10, -4) is 36.

(18) The region bounded by $y = 4x - x^2$, $x = 1$, $x = 3$ and X-axis is divided into two parts with equal area by $x = c$. Find c.

[Ans: 2]

(19) Obtain the area of the minor segment bounded by the circle $x^2 + y^2 = a^2$ and the line $x = \frac{a}{\sqrt{2}}$.

[Ans: $\frac{a^2}{4}(\pi - 2)$]

(20) Find the area of the region bounded by $y = x^2$ and $y = 2 - x$.

[Ans: $\frac{9}{2}$]

(21) Obtain the area of the region bounded by the line through A(3, 2) and B(1, 1) and the curve $x = y^2 + y - 1$.

[Ans: $\frac{1}{6}$]

(22) Obtain the area of the region bounded by the curve $y = x^2 + 1$ and the line passing through (0, 1) and (2, 5).

[Ans: $\frac{4}{3}$]

(23) Obtain the area of the region bounded by the curves $y^2 = 4x$ and $x^2 = 4y$.

[Ans: $\frac{16}{3}$]

- (24) Obtain the area of the region bounded between the circle $x^2 + y^2 = 4$ and the parabola $y^2 = 3x$.

$$\left[\text{Ans: } \frac{1}{3}(4\pi + \sqrt{3}) \right]$$

- (25) Obtain the area of the region enclosed between the parabolas $y = 6x - x^2$ and $y = x^2 - 2x$.

$$\left[\text{Ans: } \frac{64}{3} \right]$$

- (26) Obtain the volume of the solid surface generated on rotating the region bounded by the parabola $y = x^2$ and $y = 4x - x^2$, about the X-axis.

$$\left[\text{Ans: } \frac{32\pi}{3} \right]$$

- (27) Show that the volume of the segment of a sphere with radius a between two parallel planes on one side of the centre at a distance r_1 and r_2 from the centre ($r_1 < r_2$) is $\frac{\pi}{3}(r_2 - r_1)[3a^2 - (r_1^2 + r_1r_2 + r_2^2)]$.

- (28) Obtain the area of the region enclosed between $y^2 = 4x - 4$ and $y^2 = -4x + 4$.

$$\left[\text{Ans: } \frac{16}{3} \right]$$

- (29) Obtain area of the region enclosed between the parabola $y^2 = 4(x - 2)$, the line $y = x - 1$ and the X-axis.

$$\left[\text{Ans: } \frac{2}{3} \right]$$

- (30) Find the volume of the solid generated on rotating the region bounded by the curve $y = x^2 + 1$ and the line $y = 2x + 4$, about the X-axis.

$$\left[\text{Ans: } \frac{1408\pi}{15} \right]$$

(31) Find the volume of the solid generated on rotating the region bounded by $y^2 = x^3$, $x = 2$ and the X-axis about the X-axis.

[Ans: 4π]

(32) Find the volume of the solid generated on rotating the region bounded by the curve $y = a \left(\sin x + \frac{\sin 3x}{3} \right)$, the X-axis and the lines $x = 0$ and $x = \pi$ about the X-axis.

[Ans: $\frac{5\pi^2 a^2}{9}$]

(33) Find the common area enclosed between the ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ($a > b$).

[Ans: $2ab \left(\pi - 2 \sin^{-1} \frac{a}{\sqrt{a^2 + b^2}} \right)$]