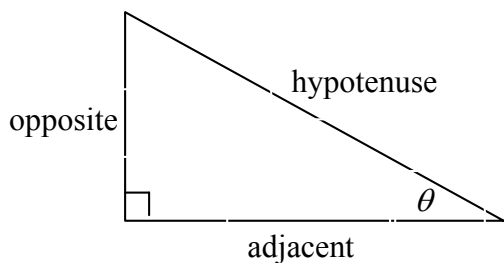


Definition of the Trig Functions

Right triangle definition

For this definition we assume that

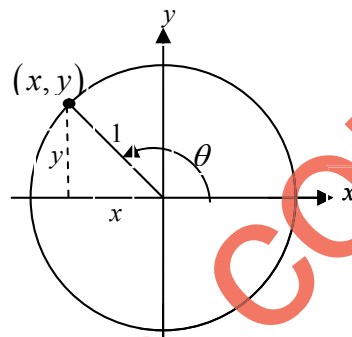
$$0 < \theta < \frac{\pi}{2} \text{ or } 0^\circ < \theta < 90^\circ.$$



$$\begin{aligned} \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}} \end{aligned}$$

Unit circle definition

For this definition θ is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{1} = y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= \frac{x}{1} = x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$$\begin{aligned} \sin \theta, & \theta \text{ can be any angle} \\ \cos \theta, & \theta \text{ can be any angle} \\ \tan \theta, & \theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \csc \theta, & \theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \sec \theta, & \theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \cot \theta, & \theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \end{aligned}$$

Range

The range is all possible values to get out of the function.

$$\begin{aligned} -1 \leq \sin \theta \leq 1 & \quad \csc \theta \geq 1 \text{ and } \csc \theta \leq -1 \\ -1 \leq \cos \theta \leq 1 & \quad \sec \theta \geq 1 \text{ and } \sec \theta \leq -1 \\ -\infty \leq \tan \theta \leq \infty & \quad -\infty \leq \cot \theta \leq \infty \end{aligned}$$

Period

The period of a function is the number, T , such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\begin{aligned} \sin(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cos(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \tan(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \\ \csc(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \sec(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cot(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \end{aligned}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \qquad \sin \theta = \frac{1}{\csc \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \qquad \cos \theta = \frac{1}{\sec \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} \qquad \tan \theta = \frac{1}{\cot \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin \theta \qquad \csc(-\theta) = -\csc \theta$$

$$\cos(-\theta) = \cos \theta \qquad \sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta \qquad \cot(-\theta) = -\cot \theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin \theta \qquad \csc(\theta + 2\pi n) = \csc \theta$$

$$\cos(\theta + 2\pi n) = \cos \theta \qquad \sec(\theta + 2\pi n) = \sec \theta$$

$$\tan(\theta + \pi n) = \tan \theta \qquad \cot(\theta + \pi n) = \cot \theta$$

Double Angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\sin \alpha - \sin \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$

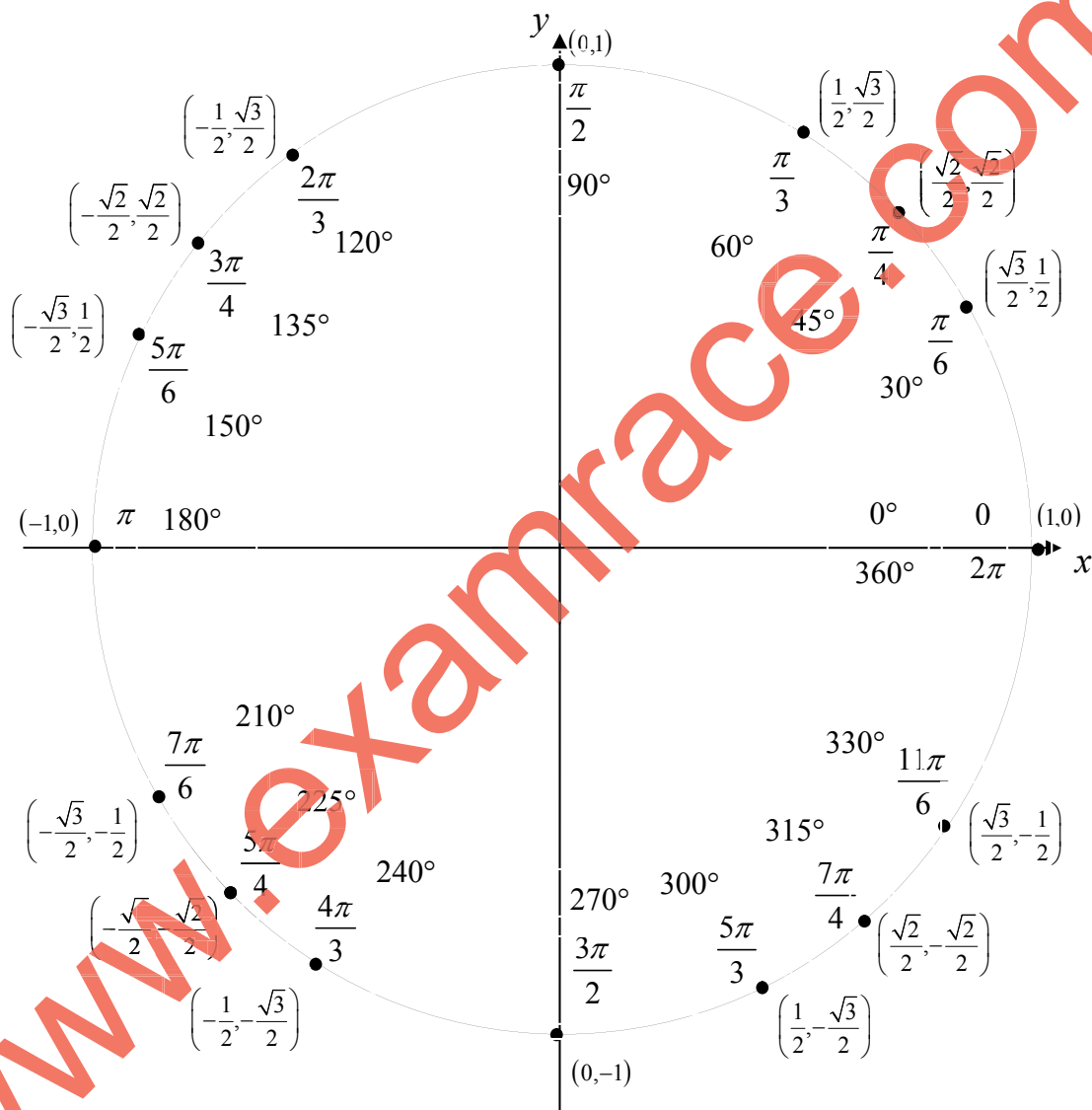
Cofunction Formulas

$$\sin \left(\frac{\pi}{2} - \theta \right) = \cos \theta \qquad \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta$$

$$\csc \left(\frac{\pi}{2} - \theta \right) = \sec \theta \qquad \sec \left(\frac{\pi}{2} - \theta \right) = \csc \theta$$

$$\tan \left(\frac{\pi}{2} - \theta \right) = \cot \theta \qquad \cot \left(\frac{\pi}{2} - \theta \right) = \tan \theta$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos \theta = x$ and $\sin \theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \quad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Inverse Properties

$$\cos(\cos^{-1}(x)) = x \quad \cos^{-1}(\cos(\theta)) = \theta$$

$$\sin(\sin^{-1}(x)) = x \quad \sin^{-1}(\sin(\theta)) = \theta$$

$$\tan(\tan^{-1}(x)) = x \quad \tan^{-1}(\tan(\theta)) = \theta$$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

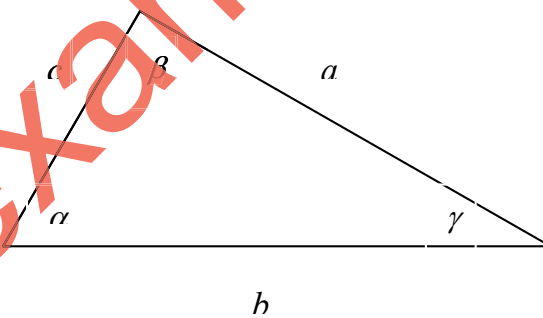
Alternate Notation

$$\sin^{-1} x = \arcsin x$$

$$\cos^{-1} x = \arccos x$$

$$\tan^{-1} x = \text{arctan } x$$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Law of Cosines

$$c^2 = b^2 + a^2 - 2ab \cos \gamma$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2} \gamma}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(\alpha - \beta)}{\tan \frac{1}{2}(\alpha + \beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(\beta - \gamma)}{\tan \frac{1}{2}(\beta + \gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(\alpha - \gamma)}{\tan \frac{1}{2}(\alpha + \gamma)}$$