IIT-JEE 2012

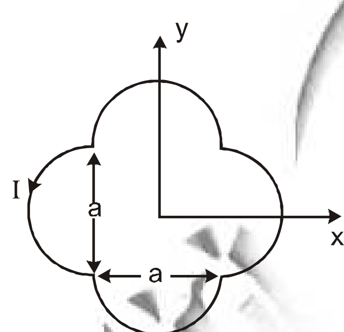
PAPER - 2

PART - I : PHYSICS

SECTION - I : Single Correct Answer Type

This section contains 8 multiple choice questions, Each question has four choices, (A), (B), (C) and (D) out of which ONLY ONE is correct.

1. A loop carrying current I lies in the x-y plane as shown in the figure. the unit vector $\hat{k}$ is coming out of the plane of the paper. the magnetic moment of the current loop is :



(A) $a^2 I\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (C) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (D) $(2\pi+1)a^2 I\hat{k}$

Ans. **(B)**

Sol. Area = $a^2 + 4 \times \frac{\pi\left(\frac{a}{2}\right)^2}{2}$

$= a^2 + \frac{\pi a^2}{2}$

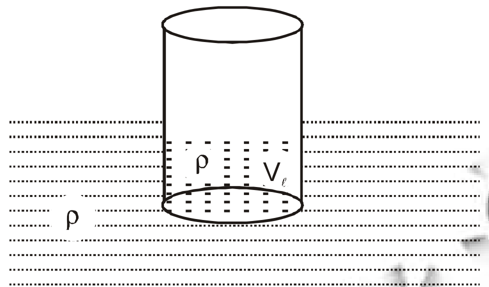
$A = \left(1+\frac{\pi}{2}\right) a^2 \hat{k}$

2. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

(A) more than half filled if ρ_c is less than 0.5
(B) more than half filled if ρ_c is less than 1.0
(C) half filled if ρ_c is less than 0.5
(D) less than half filled if ρ_c is less than 0.5

Ans. **(D)**

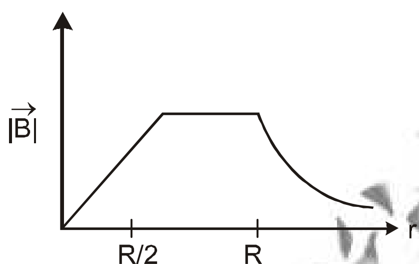
Sol. $mg + \rho \times V_\ell \times g = \frac{V_0}{2} \times \rho \times g$

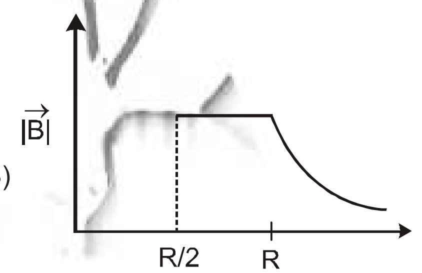


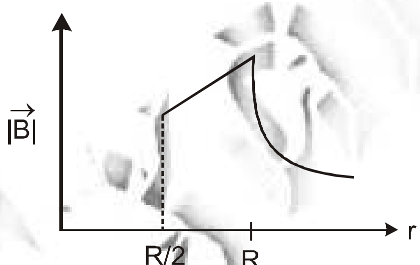
$V_\ell = \frac{V_0}{2} - \frac{m}{\rho}$

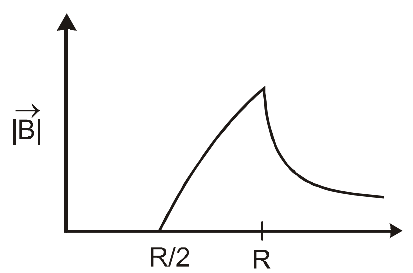
so $V_\ell < \frac{V_0}{2}$

3. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along is length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by :

(A)


(B)


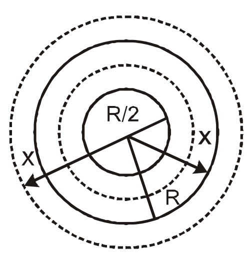
(C)


(D)


Ans. **(D)**

Sol. **Case-I** $x < \frac{R}{2}$

$|B| = 0$



Case-II $\frac{R}{2} \le x < R$

$\oint \vec{B}.d\vec{\ell} = \mu_0 I$

$|B|\, 2\pi x = \mu_0 \left[\pi x^2 - \pi\left(\frac{R}{2}\right)^2\right] J$

$|B| = \frac{\mu_0 J}{2x}\left(x^2 - \frac{R^2}{4}\right)$

Case-III $x \geq R$

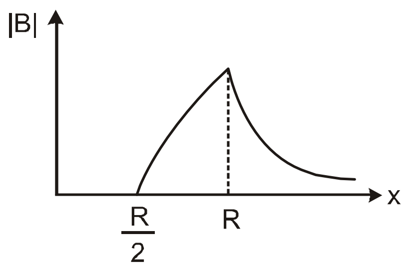
$$\int \vec{B}.\, d\vec{\ell} = \mu_0 I$$

$$|B|\, 2\pi x = \mu_0 \left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right] J$$

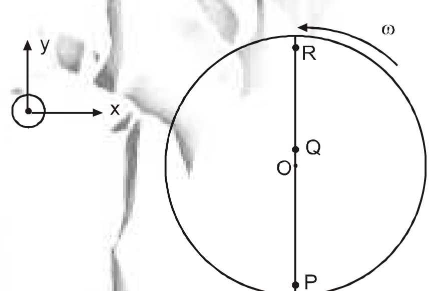
$$|B| = \frac{\mu_0 J}{2x}\ \frac{3}{2}\ R^2$$

$$|B| = \frac{3\mu_0 J R^2}{8x}$$

so



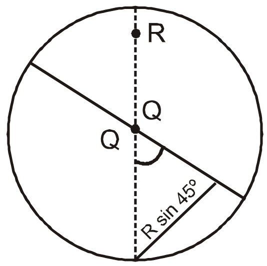
4. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc completed $\frac{1}{8}$ rotation. (ii) their range is less than half disc radius, and (iii) ω remains constant throughout . Then



(A) P lands in the shaded region and Q in the unshaded region
(B) P lands in the unshaded region and Q in the shaded region
(C) Both P and Q land in the unshaded region
(D) Both P and Q land in the shaded region

Ans. **(A)**

Sol.



To reach the unshaded portion particle P needs to travel horizontal range greater than R sin 45° or 0.7 R) but its range is less than $\frac{R}{2}$. So It will fall on shaded portion.

Q is near to origin,its velocity will be nearly along QR so its will fall in unshaded portion.
So, (A)

5. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4cm . The distance frequency of the tuning for k is 512 Hz. The air temperature is 38°C in which the ˙ peed of ˙ ound is 336 m/s. The zero of the meter scale coincides with the top and of the Resonance column. When first resonance occurs, the reading of the water level in the column is

(A) 14.0 (B) 15.2 (C) 16.4 (D) 17.6

Ans. **(B)**

Sol. $\frac{V}{4(\ell + e)} = f$

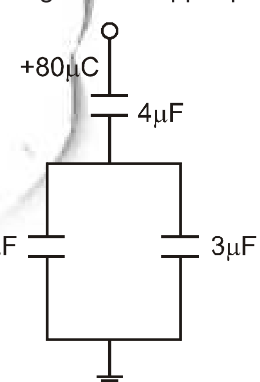
$\Rightarrow \ell + e = \frac{V}{4f}$

$\Rightarrow \ell = \frac{V}{4f} - e$

here e = (0.6)r = (0.6) (2) = 1.2 cm

so $\ell = \frac{336 \times 10^2}{4 \times 512} - 1.2 = 15.2$ ˇ"

6. In the given circuit, a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is :



(A) +32 μC (B) +40 μC (C) +48 μC (D) +80 μC

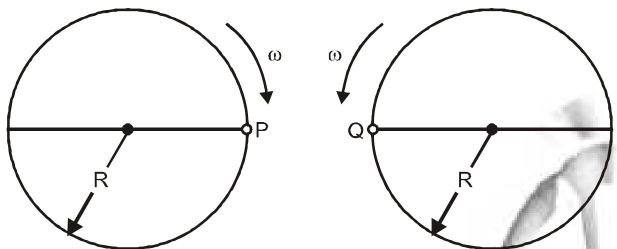
Ans. **(C)**

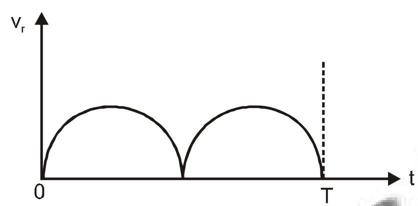
Sol. $q_3 = \frac{C_3}{C_2 + C_3}.Q$

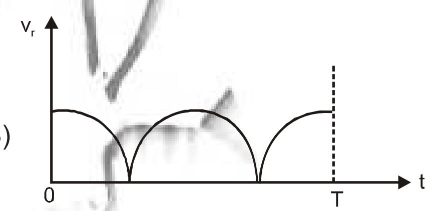
$q_3 = \frac{3}{3+2} \times 80 = \frac{3}{5} \times 80$

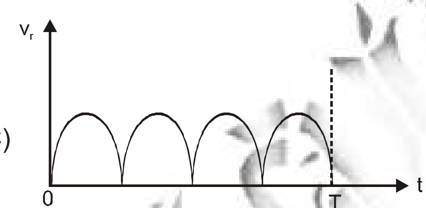
$= 48\ \mu C$

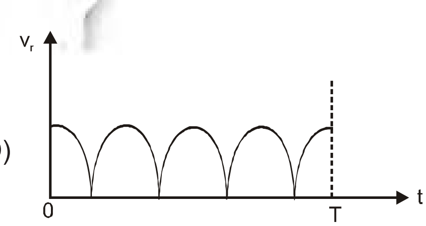
7. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The disc are in the same horizontal plane.At time t = 0 , the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. as function of times best represented by



(A)


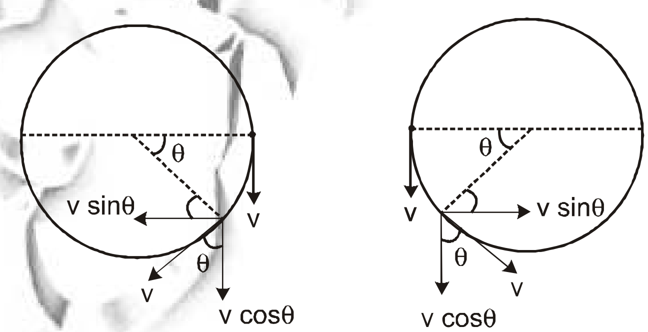
(B)


(C)


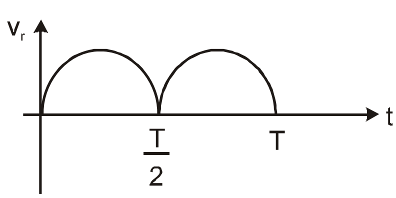
(D)


Ans. **(A)**

Sol.



$v_r = |2\ v \sin\theta)|$
$= |2v \sin \omega t)|$



8. Two moles of ideal helium gas are in a rubber balloon at 30° C .The balloon i˙ °ully ˆ(˝andable and ˇan be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C.The %" ount of heat required in raising the tˆ" perature i˙ near&' take R = 8.31 , -" ol.K)

(A) 62J (B) 104 J (C) 124 J (D) 208 J

Ans. **(D)**

Sol. $\Delta Q = nC_P \Delta T$

$$= 2\left(\frac{f}{2}R + R\right)\Delta T$$

$$= 2\left[\frac{3}{2}R + R\right] \times 5$$

$$= 2 \times \frac{5}{2} \times 8.31 \times 5$$

$= 208$ J

SECTION – II : Paragraph Type

This section contains **6 multiple choice questions** relating to **three paragraphs** with **two questions on each paragraph.** Each question has four choices (A), (B) (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 9 and 10

The β- decay process, discovered around 1900, is basically the decay of a neutron (n), In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. therefore, considering the decay of a neutron as a tro-body dcay process, it was predicted theoretically that thekinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e. $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the lectron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

9. What is the maximum energy of the anti-neutrino ?

(A) Zero (B) Much less than 0.8×10^6 eV

(C) Nearly 0.8×10^6 eV (D) Much larger than 0.8×10^6 eV

Ans. **(C)**

Sol. KE_{max} of β^-

$Q = 0.8 \times 10^6$ eV

$$KE_P + KE_{\beta^-} + KE_{\bar{\nu}} = Q$$

KE_P is almost zero

When $KE_{\beta^-} = 0$

then $KE_{\bar{\nu}} = Q - K/_P$

$\cong Q$

10. If the anti-neutrino had a mass of $3eV/c^2$ (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron ?

(A) $0 \le K \le 0.8 \times 10^6$ eV (B) 3.0 eV $\le K \le 0.8 \times 10^6$ eV

(C) 3.0 eV $\le K < 0.8 \times 10^6$ eV (D) $0 \le K < 0.8 \times 10^6$ eV

Ans. **(D)**

Sol. $0 \le KE_{\beta^-} \le Q - KE_P - KE_{\bar{\nu}}$

$0 \le KE_{\beta^-} < Q$

Paragraph for Question 11 and 12

Most materials have therefractive index, n > 1. So, when a light ray from air enters a naturally occurring material, then by Snells' law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, n = $\left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$ where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

11. Choose the correct statement.

(A) The speed of light in the meta-material is v = c|n|

(B) The speed of light in the meta-material is v = $\frac{c}{|n|}$

(C) The speed of light in the meta-material is v = c.

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air.

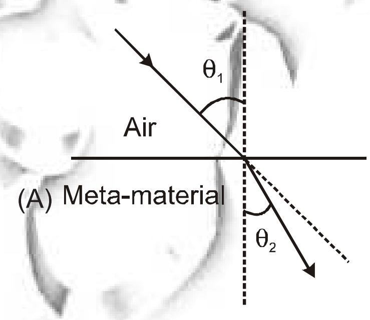
Ans. **(B)**

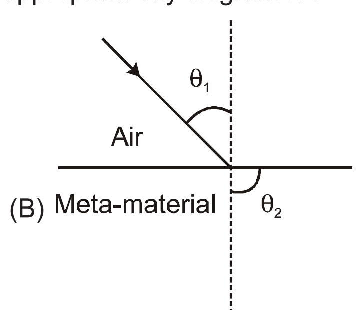
Sol. n = $\frac{c}{v}$

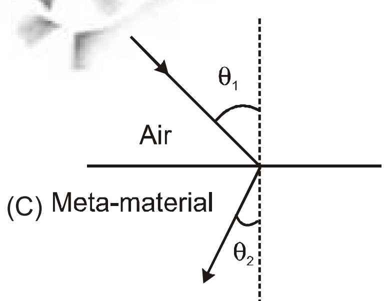
for metamaterials

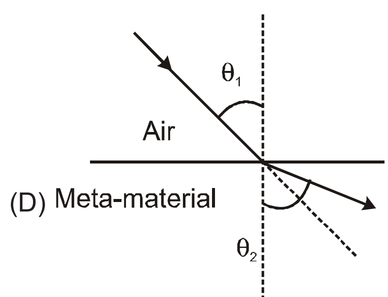
v = $\frac{c}{|n|}$

12. For light incident from air on a meta-material, the appropriate ray diagram is :









Ans. **(C)**

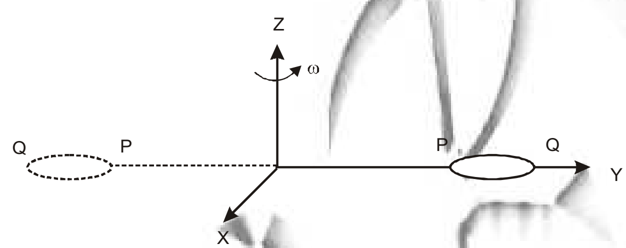
Sol. **(C)** Meta material has a negative refractive index

$\therefore$ (C) $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 \Rightarrow$ n_2 is negative

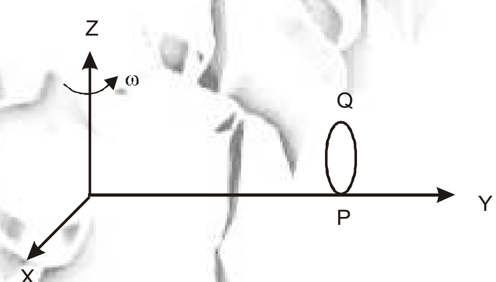
$\therefore$ θ_2 negative

Paragraph for Q. No. 13-14

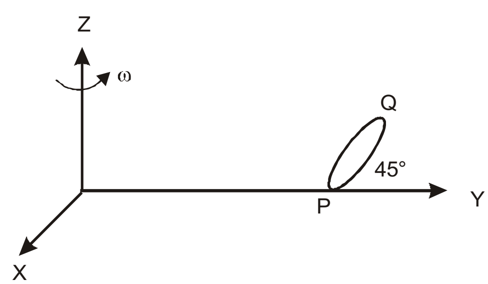
The general motion of a rigid body can be considered to be a combination of (i) a motioon --- centre of mass about an axis, and (ii) its motion about an instantanneous axis passing through center of mass. These axes need not be stationary. Consider, for example, a thin uniform welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. Where disc-stick system is rotated about the origin ona horizontal frictionless plane with angular sp--- ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis pass through its centre of mass (as is seen from the changed orientation of points P and Q). Both the motions have the same angular speed ω in the case.



Now consider two similar systems as shown in the figure: case (a) the disc with its face ver--- and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with (0 ˝lane its horizontal ˛ iamˆ ter parallel to x-axis. In both the cases, the disc is weleded at point P, and systems are rotated with constant angular speed ω about the z-axis.



Case (a)



Case (b)

13. Which of the following statement regarding the angular speed about the istantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2}\,\omega$ for boht the cases

(B) it is ω for case (a); and $\frac{w}{\sqrt{2}}$ for case (b).

(C) It is ω for case (a); and $\sqrt{2}\,\omega$ for case (b)

(D) It is ω for both the cases

Ans. **(D)**

14. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is vertical for both the cases (a) and (b).

(B) It is verticle for case (a); and is at 45° to the (01 ˝lane %nd lies in the ˝lane of the ˛ is˘ °or ˘%ˆ b)

(C) It is horizontal ofr case (a); and is at 45° to the (01 plane and is nor" %l to the ˝lane o° the dis˘ °or ˘%ˆ (b).

(D) It is vertical of case (a); and is at 45° to the (01 plane and is nor" %l to the ˝lane o° the dis˘ °or ˘%ˆ b).

Ans. **(A)**

SECTION – III : Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

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15. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane form the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement (s) is (are) correct?

(A) Both cylinders P and Q reach the ground at the same time

(B) Cylinder P has larger linear acceleration than cylinder Q.

(C) Both cylinder Q reaches the ground with same translational kinetic energy.

(D) Cylinder Q reaches the ground with larger angular speed.

Ans. **(D)**

Sol. $I_P > I_Q$

$a_P = \dfrac{g\sin\theta}{I_P + mR^2}$

$a_Q = \dfrac{g\sin\theta}{I_Q + mR^2}$

$a_P < a_Q \Rightarrow V = u + at \Rightarrow t \propto \dfrac{1}{a}$

$t_P > t_Q$

$V^2 = u^2 + 2as \Rightarrow v \propto a \Rightarrow V_P < V_Q$

Translational K.E. $= \dfrac{1}{2}mV^2 \Rightarrow TR\ KE_P < TR\ KE_Q$

$V = \omega R \Rightarrow \omega \propto V \Rightarrow \omega_P < \omega_Q$

16. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement (s) is (are) :

(A) the emf induced in the loop is zero if the current is constant.

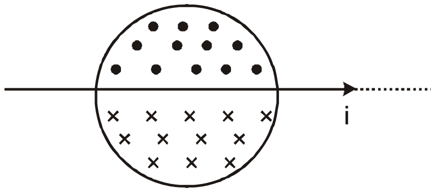
(B) The emf induced in the loop is finite if the current is constant.

(C) The emf induced in the loop is zero if the current decreases at a steady rate.

(D) Theemf induced in the loop is finite if the current decreases at a steady rate.

Ans. **(A,C)**

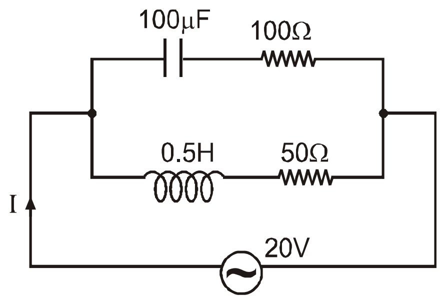
Sol.



$(\phi)_{loop}$ = 0 for all cases

so induced emf = 0

17. In the given circuit, the AC source has ω = 100 rad/s. considering the inductor and capacitor to be ideal, the correct choice (s) is(are)



(A) The current through the circuit, I is 0.3 A

(B) The current through the circuit, I is $0.3\sqrt{2}$ A.

(C) The voltage across 100Ω resistor = $10\sqrt{2}$ V

(D) The voltage across 50Ω resistor = 10V

Ans. **(A,C)**

Sol. C = 100 μF, $\frac{1}{\omega C} = \frac{1}{(100)(100\times10^{-6})}$

$X_C = 100\ \Omega$, $X_L = \omega L = (100)(.5) = 50\ \Omega$

$Z_1 = \sqrt{x_C^2 + 100^2} = 100\sqrt{2}\Omega$

$Z_2 = \sqrt{x_L^2 + 50^2} = \sqrt{50^2 + 50^2}$

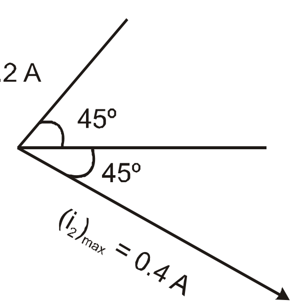
$= 50\sqrt{2}$

$\varepsilon = 20\sqrt{2}\sin\omega t$

$i_1 = \frac{20\sqrt{2}}{100\sqrt{2}}\sin(\omega t + \pi/4)$

$i_1 = \frac{1}{5}\sin(\omega t + \pi/4)$

$I_2 = \frac{20\sqrt{2}}{50\sqrt{2}}\sin(\omega t - \pi/4)$



$I = \sqrt{(.2)^2 + (.4)^2}$

$= (.2)\ \sqrt{1+4}$

$= \frac{1}{5}\ \sqrt{5}\ = \frac{1}{\sqrt{5}}$

$(I)_{rms} = \frac{1}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\approx 0.3\,A$

$(V_{100\Omega})_{rms} = (I_1)_{rms}) \times 100$

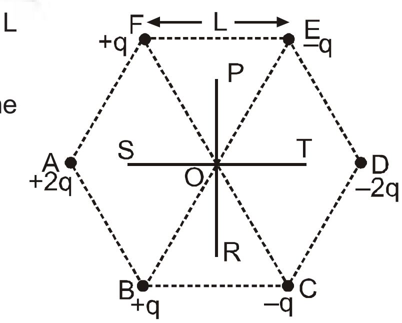
$= \left(\frac{0.2}{\sqrt{2}}\right) \times$ ˜00 $= \frac{20}{\sqrt{2}}$

$= 10\ \sqrt{2}\ V$

$V_{50\Omega})_{rms} = \left(\frac{0.4}{\sqrt{2}}\right) \times 50$

$= \frac{20}{\sqrt{2}} = 10\sqrt{2}\ V$

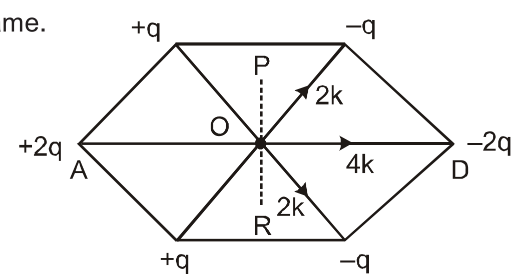
18. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement (s) is (are) correct ?



(A) the elecric field at O is 6K along OD
(B) The potential at O is zero
(C) The potential at all points on the line PR is same
(D) The potential at all points on the line ST is same.

Ans. **(A,B,C)**

Sol. $E_0 = 6\ K$ (along OD)
$V_0 = 0$
Potential on line PR is zero
Ans. (A), (B), (C)



19. Two spherical planets P and Q have the same unfirom density ρ, masses M_P and M_Q, an surface areas A and 4A, respectively. A spherical planet R also has unfirom density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V respectivley. Then

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P / V_Q = \frac{1}{2}$

Ans. **(B,D)**

Sol. $V_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2.G\rho.\frac{4}{3}\pi R^3}{R}} = \sqrt{\frac{4G\rho}{3}}\ R$

$V_{es} \propto R$

Sarface area of $P = A = 4\pi R_P^2$

Surface area of $Q = 4A = 4\pi R_Q^2$

$\Rightarrow R_Q = 2R_p$

mass R is $M_R = M_P + M_Q$

$$\rho\frac{4}{3}\pi R_R^3 = \rho\frac{4}{3}\pi R_P^3 + \rho\frac{4}{3}\pi R_Q^3$$

$\Rightarrow R_R^3 = R_P^3 + R_Q^3$

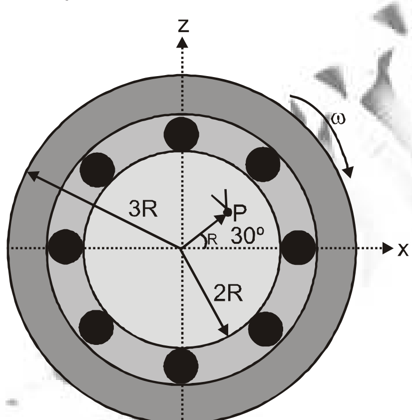
$= 9R_P^3$

$R_R = 9^{1/3} R_P \Rightarrow R_R > R_Q > R_P$

Therefore $V_R > V_Q > V_P$

$$\frac{V_R}{V_P} = 9^{1/3} \quad \text{and} \quad \frac{V_P}{V_Q} = \frac{1}{2}$$

20. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated b frictionaless ball bearings. The system is in the x-z plane. The point P on the inner disc is at distance R from the origin, where OP makes an angle of 30º with the horizontal. Then with respect to the horizontal surface,



(A) the point O has linear velocity $3R\omega\hat{i}$.

(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$

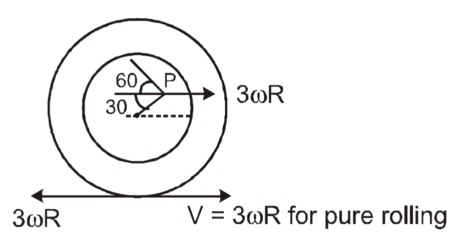
(C) the point P has linear velocity $\frac{13}{4}Rw\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) The point P has a linear velocity $\left(3 - \frac{\sqrt{3}}{4}\right)Rw\hat{i} + \frac{1}{4}Rw\hat{k}$.

Ans. **(A,B)**

Sol. $V_0 = 3\omega R\,\hat{i}$

$V_P\ (3\omega R - \frac{\omega R}{2}\cos 60º)\ \hat{i} + \frac{\omega R}{2}\sin 60\ \hat{j}$



$= \frac{11\omega R}{4}\hat{i} + \frac{\sqrt{3}\omega R}{4}\hat{i}$